# **PyBaMM Documentation**

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Python Battery Mathematical Modelling (PyBAMM) solves continuum models for batteries, using both numerical methods and asymptotic analysis.

PyBaMM is hosted on GitHub. This page provides the API, or developer documentation for pybamm.

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## CHAPTER 1

Quickstart

PyBaMM is available on GNU/Linux, MacOS and Windows.

## 1.1 Using pip

## 1.1.1 GNU/Linux and Windows

pip install pybamm

### 1.1.2 macOS

brew install sundials && pip install pybamm

## 1.2 Using conda

PyBaMM is available as a conda package through the conda-forge channel.

conda install -c conda-forge pybamm

## 1.3 Optional solvers

On GNU/Linux and MacOS, an optional scikits.odes -based solver is available, see Optional - scikits.odes solver.

## CHAPTER 2

Installation

## 2.1 GNU-Linux & MacOS

## Contents

- GNU-Linux & MacOS
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## 2.1.1 Prerequisites

To use and/or contribute to PyBaMM, you must have Python 3.6 or 3.7 installed (note that 3.8 is not yet supported). To install Python 3 on Debian-based distribution (Debian, Ubuntu, Linux mint), open a terminal and run

```
sudo apt update
sudo apt install python3
```

On Fedora or CentOS, you can use DNF or Yum. For example

```
sudo dnf install python3
```

On Mac OS distributions, you can use homebrew. First *install* "brew" <a href="https://docs.python-guide.org/starting/">https://docs.python-guide.org/starting/</a> install3/osx/>"\_\_:

```
ruby -e "$(curl -fsSL https://raw.githubusercontent.com/Homebrew/install/master/
→install)"
```

then follow instructions in link on adding brew to path, and run

```
brew install python3
```

## 2.1.2 Install PyBaMM

#### **User install**

We recommend to install PyBaMM within a virtual environment, in order not to alter any distribution python files. First, make sure you are using python 3.6 or 3.7. To create a virtual environment env within your current directory type:

```
virtualenv env
```

You can then "activate" the environment using:

```
source env/bin/activate
```

Now all the calls to pip described below will install PyBaMM and its dependencies into the environment env. When you are ready to exit the environment and go back to your original system, just type:

```
deactivate
```

PyBaMM can be installed via pip. On macOS, it is necessary to install the SUNDIALS library beforehand.

#### **GNU/Linux and Windows**

```
pip install pybamm
```

#### macOS

```
brew install sundials pip install pybamm
```

PyBaMM's dependencies (such as numpy, scipy, etc) will be installed automatically when you install PyBaMM using pip.

For an introduction to virtual environments, see (https://realpython.com/python-virtual-environments-a-primer/).

#### Optional - scikits.odes solver

Users can install scikits.odes in order to use the wrapped SUNDIALS ODE and DAE solvers. Currently, only GNU/Linux and macOS are supported.

#### **GNU/Linux**

```
apt install libopenblas-dev pybamm_install_odes --install-sundials
```

The pybamm\_install\_odes command is installed with PyBaMM. It automatically downloads and installs the SUNDIALS library on your system (under ~/.local), before installing sckits.odes (by running pip install scikits.odes).

#### macOS

```
pip install scikits.odes
```

Assuming that the SUNDIALS were installed as described above.

#### **Developer install**

If you wish to contribute to PyBaMM, you should get the latest version from the GitHub repository. To do so, you must have Git and graphviz installed. For instance run

```
sudo apt install git graphviz
```

on Debian-based distributions, or

```
brew install git graphviz
```

on Mac OS.

To install PyBaMM, the first step is to get the code by cloning this repository

```
git clone https://github.com/pybamm-team/PyBaMM.git cd PyBaMM
```

Then, to install PyBaMM as a developer, type

```
pip install -e .[dev,docs]
```

**KLU** sparse solver If you wish so simulate large systems such as the 2+1D models, we recommend employing a sparse solver. PyBaMM currently offers a direct interface to the sparse KLU solver within Sundials, but it is unlikely to be installed as you may not have all the dependencies available. If you wish to install the KLU from the PyBaMM sources, see *the instructions for compiling the KLU sparse solver*.

To check whether PyBaMM has installed properly, you can run the tests:

```
python3 run-tests.py --unit
```

Before you start contributing to PyBaMM, please read the contributing guidelines.

## 2.1.3 Uninstall PyBaMM

PyBaMM can be uninstalled by running

```
pip uninstall pybamm
```

in your virtual environment.

## 2.1.4 Troubleshooting

Problem: I've made edits to source files in PyBaMM, but these are not being used when I run my Python script.

**Solution:** Make sure you have installed PyBaMM using the -e flag, i.e. pip install -e .. This sets the installed location of the source files to your current directory.

**Problem:** When running python run-tests.py --quick, gives error FileNotFoundError: [Errno 2] No such file or directory: 'flake8': 'flake8.

**Solution:** make sure you have included the [dev,docs] flags when you pip installed PyBaMM, i.e. pip install -e .[dev,docs]

**Problem:** Errors when solving model ValueError: Integrator name ida does not exsist, or ValueError: Integrator name cvode does not exsist.

**Solution:** This could mean that you have not installed scikits.odes correctly, check the instructions given above and make sure each command was successful.

One possibility is that you have not set your LD\_LIBRARY\_PATH to point to the sundials library, type echo \$LD\_LIBRARY\_PATH and make sure one of the directories printed out corresponds to where the sundials libraries are located.

Another common reason is that you forget to install a BLAS library such as OpenBLAS before installing sundials. Check the cmake output when you configured Sundials, it might say:

```
-- A library with BLAS API not found. Please specify library location.
-- LAPACK requires BLAS
```

If this is the case, on a Debian or Ubuntu system you can install OpenBLAS using sudo apt-get install libopenblas-dev (or brew install openblas for Mac OS) and then re-install sundials using the instructions above.

## 2.2 Windows

- Windows
  - Prerequisites
  - Install PyBaMM
    - \* User install
  - Uninstall PyBaMM
  - Installation using WSL

### 2.2.1 Prerequisites

To use and/or contribute to PyBaMM, you must have Python 3.6 or 3.7 installed (note that 3.8 is not yet supported).

To install Python 3 download the installation files from Python's website. Make sure to tick the box on Add Python 3.X to PATH. For more detailed instructions please see the official Python on Windows guide.

## 2.2.2 Install PyBaMM

#### **User install**

Launch the Command Prompt and go to the directory where you want to install PyBaMM. You can find a reminder of how to navigate the terminal here.

We recommend to install PyBaMM within a virtual environment, in order not to alter any distribution python files.

To create a virtual environment env within your current directory type:

```
python -m venv env
```

You can then "activate" the environment using:

```
env\Scripts\activate.bat
```

Now all the calls to pip described below will install PyBaMM and its dependencies into the environment env. When you are ready to exit the environment and go back to your original system, just type:

deactivate

PyBaMM can be installed via pip:

```
pip install pybamm
```

PyBaMM's dependencies (such as numpy, scipy, etc) will be installed automatically when you install PyBaMM using pip.

For an introduction to virtual environments, see (https://realpython.com/python-virtual-environments-a-primer/).

## 2.2.3 Uninstall PyBaMM

PyBaMM can be uninstalled by running

```
pip uninstall pybamm
```

in your virtual environment.

## 2.2.4 Installation using WSL

If you want to install the optional PyBaMM solvers, you have to use the Windows Subsystem for Linux (WSL). You can find the installation instructions here.

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## 2.3 Windows Subsystem for Linux (WSL)

We recommend the use of Windows Subsystem for Linux (WSL) to install PyBaMM, see the instructions below to get PyBaMM working using Windows, WSL and VSCode.

#### **Contents**

- Windows Subsystem for Linux (WSL)
  - Install WSL
  - Install PyBaMM
  - Use Visual Studio Code to run PyBaMM

#### 2.3.1 Install WSL

Follow the instructions from Microsoft here. When given the option, choose the Ubuntu 18.04 LTS distribution to install. Don't forget to initialise the Ubuntu installation using the instructions given here.

## 2.3.2 Install PyBaMM

Open a terminal window in your installed Ubuntu distribution by selecting "Ubuntu" from the start menu. This should give you a bash prompt in your home directory.

To download the PyBaMM source code, you first need to install git, which you can do by typing

```
sudo apt install git-core
```

For easier integration with WSL, we recommend that you install PyBaMM in your Windows Documents folder, for example by first navigating to

```
$ cd /mnt/c/Users/USER_NAME/Documents
```

where USER\_NAME is your username. Exact path to Windows documents may vary. Now use git to clone the PyBaMM repository:

```
git clone https://github.com/pybamm-team/PyBaMM.git
```

This will create a new directly called PyBaMM, you can move to this directory in bash using the cd command:

```
cd PyBaMM
```

If you are unfamiliar with the linux command line, you might find it useful to work through this tutorial provided by Ubuntu.

Now head over and follow the installation instructions for PyBaMM for linux here.

## 2.3.3 Use Visual Studio Code to run PyBaMM

You will probably want to use a native Windows IDE such as Visual Studio Code or the full Microsoft Visual Studio IDE. Both of these packages can connect to WSL so that you can write python code in a native windows environment,

while at the same time using WSL to run the code using your installed Ubuntu distribution. The following instructions assume that you are using Visual Studio Code.

First, setup VSCode to run within the PyBaMM directory that you created above, using the instructions provided here.

Once you have opened the PyBaMM folder in vscode, use the Extensions panel to install the Python extension from Microsoft. Note that extensions are either installed on the Windows (Local) or on in WSL (WSL:Ubuntu), so even if you have used VSCode previously with the Python extension, you probably haven't installed it in WSL. Make sure to reload after installing the Python extension so that it is available.

If you have installed PyBaMM into the virtual environment env as in the PyBaMM linux install guide, then VSCode should automatically start using this environment and you should see something similar to "Python 3.6.8 64-bit ('env': venv)" in the bottom bar.

To test that vscode can run a PyBaMM script, navigate to the examples/scripts folder and right click on the create-model.py script. Select "Run current file in Python Interactive Window". This should run the script, which sets up and solves a model of SEI thickness using PyBaMM. You should see a plot of SEI thickness versus time pop up in the interactive window.

The Python Interactive Window in VSCode can be used to view plots, but is restricted in functionality and cannot, for example, launch separate windows to show plot. To setup an xserver on windows and use this to launch windows for plotting, follow these instructions:

- 1. Install VcXsrv from here.
- 2. Set the display port in the WSL command-line: echo "export DISPLAY=localhost:0.0" >> ~/. bashrc
- 3. Install python3-tk in WSL: sudo apt-get install python3-tk
- 4. Set the matplotlib backend to TKAgg in WSL: echo "backend: TKAgg" >> ~/.config/matplotlib/matplotlibrc
- 5. Before running the code, just launch XLaunch (with the default settings) from within Windows. Then the code works as usual.

## 2.4 PyBaMM developer install - The KLU sparse solver

If you wish to try a different DAE solver, PyBaMM currently offers a direct interface to the sparse KLU solver within Sundials. This solver comes as a C++ python extension module. Therefore, when installing PyBaMM from source (e.g. from the GitHub repository), the KLU sparse solver module must be compiled. Running pip install . or python setup.py install in the PyBaMM directory will result in a attempt to compile the KLU module.

Note that if CMake of pybind11 are not found (see below), the installation of PyBaMM will carry on, however skipping the compilation of the idaklu module. This allows developers that are not interested in the KLU module to install PyBaMM from source without having to install the required dependencies.

To build the KLU solver, the following dependencies are required:

- A C++ compiler (e.g. g++)
- A Fortran compiler (e.g. gfortran)
- The python 3 header files
- CMake
- A BLAS implementation (e.g. openblas)
- pybind11
- sundials

SuiteSparse

The first four should be available through your favourite package manager. On Debian-based GNU/Linux distributions:

```
apt update
apt install python3-dev gcc gfortran cmake libopenblas-dev
```

## 2.4.1 pybind11

The pybind11 source directory should be located in the PyBaMM project directory at the time of compilation. Simply clone the GitHub repository, for example:

```
# In the PyBaMM project dir (next to setup.py)
git clone https://github.com/pybind/pybind11.git
```

## 2.4.2 SuiteSparse and sundials

#### Method 1 - Using the convenience script

The PyBaMM repository contains a script scripts/setup\_KLU\_module\_build.py that automatically downloads, extracts, compiles and installs the two libraries.

First install the Python wget module

```
pip install wget
```

Then execute the script

```
# In the PyBaMM project dir (next to setup.py)
python scripts/setup_KLU_module_build.py
```

The above will install the required component of SuiteSparse and Sundials in your home directory under  $\sim$ /.local/. Note that you can provide the option --install-dir=<install/path> to install both libraries to an alternative location. If <install/path> is not absolute, it will be interpreted as relative to the PyBaMM project directory.

Finally, reactivate your virtual environment by running

```
source $(VIRTUAL_ENV)/bin/activate
```

Alternatively, you update the LD\_LIBRARY\_PATH environment variable as follows

```
export LD_LIBRARY_PATH=$(HOME)/.local:$LD_LIBRARY_PATH
```

The above export statement will be ran automatically the next time you activate you python virtual environment.

If did not run the convenience script inside a python virtual environment, execute you bash config file

```
source ~/.bashrc
```

(or start a new shell).

Build files are located inside the PyBaMM project directory under KLU\_module\_deps/. Feel free to remove this directory once everything is installed correctly.

#### Method 2 - Compiling Sundials (advanced)

#### **SuiteSparse**

On most current linux distributions and macOS, a recent enough version of the suitesparse source package is available through the package manager. For instance on Fedora

```
yum install libsuitesparse-dev
```

#### **Sundials**

The PyBaMM KLU solver requires Sundials >= 4.0. Because most Linux distribution provide older versions through their respective package manager, it is recommended to build and install Sundials manually.

First, download and extract the sundials 5.0.0 source

```
\begin{tabular}{ll} wget & https://computing.llnl.gov/projects/sundials/download/sundials-5.0.0.tar.gz & tar -xvf sundials-5.0.0.tar.gz & tar -xvf sundials-5.0.tar.gz & tar -xvf sundials-5.0.t
```

Then, create a temporary build directory and navigate into it

```
mkdir build_sundials cd build_sundials
```

You can now configure the build, by running

```
cmake -DLAPACK_ENABLE=ON\
    -DSUNDIALS_INDEX_SIZE=32\
    -DBUILD_ARKODE=OFF\
    -DBUILD_CVODE=OFF\
    -DBUILD_CVODES=OFF\
    -DBUILD_IDAS=OFF\
    -DBUILD_KINSOL=OFF\
    -DEXAMPLES_ENABLE:BOOL=OFF\
    -DKLU_ENABLE=ON\
    -DKLU_INCLUDE_DIR=path/to/suitesparse/headers\
    -DKLU_LIBRARY_DIR=path/to/suitesparse/libraries\
    ../sundials-5.0.0
```

Be careful set the two variables KLU\_INCLUDE\_DIR and KLU\_LIBRARY\_DIR to the correct installation location of the SuiteSparse libary on your system. If you installed SuiteSparse through your package manager, this is likely to be something similar to:

```
-DKLU_INCLUDE_DIR=/usr/include/suitesparse\
-DKLU_LIBRARY_DIR=/usr/lib/x86_64-linux-gnu\
```

By default, Sundials will be installed on your system under /usr/local (this varies depending on the distribution). Should you wish to install sundials in a specific location, set the following variable

```
-DCMAKE_INSTALL_PREFIX=install/location\
```

Finally, build the library:

```
make install
```

You may be asked to run this command as a super-user, depending on the installation location.

#### **Alternative installation location**

By default, it is assumed that the SuiteSparse and Sundials libraries are installed in your home directory under ~/ .local. If you installed the libraries to (a) different location(s), you must set the options suitesparse-root or/and sundials-root when installing PyBaMM. Examples:

```
python setup.py install --suitesparse-root=path/to/suitesparse
```

or

```
pip install . --install-option="--sundials-root=path/to/sundials"
```

## 2.4.3 (re)Install PyBaMM to build the KLU solver

If the above dependencies are correctly installed, any of the following commands will install PyBaMM with the idaklu solver module:

```
pip install .
pip install -e .
python setup.py install
python setup.py develop
...
```

Note that it doesn't matter if pybamm is already installed. The above commands will update your exisiting installation by adding the idaklu module.

## 2.4.4 Check that the solver is correctly installed

If you install PyBaMM in editable mode using the -e pip switch or if you use the python setup.py install command, a log file will be located in the project directory (next to the setup.py file).

```
cat setup.log
020-03-24 11:33:50,645 - PyBaMM setup - INFO - Starting PyBaMM setup
2020-03-24 11:33:50,653 - PyBaMM setup - INFO - Not running on windows
2020-03-24 11:33:50,654 - PyBaMM setup - INFO - Could not find CMake. Skipping
compilation of KLU module.
2020-03-24 11:33:50,655 - PyBaMM setup - INFO - Could not find pybind11 directory (/
```

If the KLU sparse solver is correctly installed, then the following command should return True.

```
$ python -c "import pybamm; print(pybamm.have_idaklu())"
```

## CHAPTER 3

### API documentation

## 3.1 Expression Tree

## **3.1.1 Symbol**

**class** pybamm. **Symbol** (name, children=None, domain=None, auxiliary\_domains=None) Base node class for the expression tree

#### **Parameters**

- name (str) name for the node
- **children** (iterable *Symbol*, optional) children to attach to this node, default to an empty list
- **domain** (*iterable of str*, *or str*) list of domains over which the node is valid (empty list indicates the symbol is valid over all domains)
- auxiliary\_domains (dict of str) dictionary of auxiliary domains over which the node is valid (empty dictionary indicates no auxiliary domains). Keys can be "secondary" or "tertiary". The symbol is broadcast over its auxiliary domains. For example, a symbol might have domain "negative particle", secondary domain "separator" and tertiary domain "current collector" (domain="negative particle", auxiliary\_domains={"secondary": "separator", "tertiary": "current collector"}).

```
__abs__()
    return an AbsoluteValue object

__add__(other)
    return an Addition object

__ge__(other)
    return a EqualHeaviside object

__gt__(other)
    return a NotEqualHeaviside object
```

```
__init__ (name, children=None, domain=None, auxiliary_domains=None)
     Initialize self. See help(type(self)) for accurate signature.
___le__(other)
     return a Equal Heaviside object
lt (other)
    return a NotEqualHeaviside object
matmul (other)
    return a MatrixMultiplication object
 _mul___(other)
    return a Multiplication object
__neg__()
    return a Negate object
__pow__ (other)
    return a Power object
radd (other)
    return an Addition object
__repr__()
    returns the string __class__(id, name, children, domain)
rmatmul (other)
    return a MatrixMultiplication object
___rmul__(other)
     return a Multiplication object
___rpow___(other)
    return a Power object
__rsub__(other)
    return a Subtraction object
__rtruediv__(other)
    return a Division object
__str__()
    return a string representation of the node and its children
__sub__(other)
    return a Subtraction object
 _truediv__(other)
    return a Division object
auxiliary_domains
     Returns domains that are not the primary domain
children
     returns the cached children of this node.
     Note: it is assumed that children of a node are not modified after initial creation
clear_domains()
     Clear domains, bypassing checks
{\tt copy\_domains}\ (symbol)
     Copy the domains from a given symbol, bypassing checks
```

#### diff(variable)

Differentiate a symbol with respect to a variable. For any symbol that can be differentiated, return 1 if differentiating with respect to yourself, self.\_diff(variable) if variable is in the expression tree of the symbol, and zero otherwise.

**Parameters variable** (pybamm. Symbol) – The variable with respect to which to differentiate

#### domain

list of applicable domains

#### Returns

**Return type** iterable of str

#### **evaluate** (*t*=None, *y*=None, *y*\_dot=None, inputs=None, known\_evals=None)

Evaluate expression tree (wrapper to allow using dict of known values). If the dict 'known\_evals' is provided, the dict is searched for self.id; if self.id is in the keys, return that value; otherwise, evaluate using \_base\_evaluate() and add that value to known\_evals

#### **Parameters**

- t (float or numeric type, optional) time at which to evaluate (default None)
- **y** (numpy.array, optional) array with state values to evaluate when solving (default None)
- **y\_dot** (numpy.array, optional) array with time derivatives of state values to evaluate when solving (default None)
- inputs (dict, optional) dictionary of inputs to use when solving (default None)
- known\_evals (dict, optional) dictionary containing known values (default None)

#### Returns

- number or array the node evaluated at (t,y)
- known\_evals (if known\_evals input is not None) (dict) the dictionary of known values

#### evaluate\_for\_shape()

Evaluate expression tree to find its shape. For symbols that cannot be evaluated directly (e.g. *Variable* or *Parameter*), a vector of the appropriate shape is returned instead, using the symbol's domain. See <code>pybamm.Symbol.evaluate()</code>

#### evaluate\_ignoring\_errors(t=0)

Evaluates the expression. If a node exists in the tree that cannot be evaluated as a scalar or vector (e.g. Time, Parameter, Variable, StateVector), then None is returned. If there is an InputParameter in the tree then a 1 is returned. Otherwise the result of the evaluation is given.

#### See also:

evaluate() evaluate the expression

#### evaluates on edges (dimension)

Returns True if a symbol evaluates on an edge, i.e. symbol contains a gradient operator, but not a divergence operator, and is not an IndefiniteIntegral.

**Parameters dimension** (str) – The dimension (primary, secondary, etc) in which to query evaluation on edges

**Returns** Whether the symbol evaluates on edges (in the finite volume discretisation sense)

Return type bool

```
evaluates_to_number()
```

Returns True if evaluating the expression returns a number. Returns False otherwise, including if NotImplementedError or TyperError is raised. !Not to be confused with isinstance(self, pybamm.Scalar)!

See also:

```
evaluate () evaluate the expression
```

```
get_children_auxiliary_domains (children)
```

Combine auxiliary domains from children, at all levels

```
has_symbol_of_classes (symbol_classes)
```

Returns True if equation has a term of the class(es) symbol\_class.

**Parameters symbol\_classes** (pybamm class or iterable of classes) - The classes to test the symbol against

```
is_constant()
```

returns true if evaluating the expression is not dependent on t or y or u

See also:

```
evaluate () evaluate the expression
```

```
jac (variable, known jacs=None, clear domain=True)
```

Differentiate a symbol with respect to a (slice of) a StateVector or StateVectorDot. See pybamm. Jacobian.

#### name

name of the node

```
new_copy()
```

Make a new copy of a symbol, to avoid Tree corruption errors while bypassing copy.deepcopy(), which is slow.

#### orphans

Returning new copies of the children, with parents removed to avoid corrupting the expression tree internal data

#### pre\_order()

returns an iterable that steps through the tree in pre-order fashion

#### **Examples**

#### relabel tree(symbol, counter)

Finds all children of a symbol and assigns them a new id so that they can be visualised properly using the graphviz output

#### render()

print out a visual representation of the tree (this node and its children)

#### secondary domain

Helper function to get the secondary domain of a symbol

#### set id()

Set the immutable "identity" of a variable (e.g. for identifying y\_slices).

This is identical to what we'd put in a \_\_hash\_\_ function However, implementing \_\_hash\_\_ requires also implementing \_\_eq\_\_, which would then mess with loop-checking in the anytree module.

Hashing can be slow, so we set the id when we create the node, and hence only need to hash once.

#### shape

Shape of an object, found by evaluating it with appropriate t and y.

#### shape for testing

Shape of an object for cases where it cannot be evaluated directly. If a symbol cannot be evaluated directly (e.g. it is a *Variable* or *Parameter*), it is instead given an arbitrary domain-dependent shape.

#### simplify (simplified symbols=None)

Simplify the expression tree. See pybamm. Simplification.

#### size

Size of an object, found by evaluating it with appropriate t and y

#### size\_for\_testing

Size of an object, based on shape for testing

#### test shape()

Check that the discretised self has a pybamm shape, i.e. can be evaluated

Raises pybamm. ShapeError - If the shape of the object cannot be found

to\_casadi (t=None, y=None, y\_dot=None, inputs=None, casadi\_symbols=None)

Convert the expression tree to a CasADi expression tree. See pybamm. CasadiConverter.

#### visualise (filename)

Produces a .png file of the tree (this node and its children) with the name filename

**Parameters filename** (str) – filename to output, must end in ".png"

#### 3.1.2 Parameter

```
class pybamm.Parameter(name, domain=[])
```

A node in the expression tree representing a parameter

This node will be replaced by a Scalar node by :class'.Parameter'

#### **Parameters**

- name (str) name of the node
- domain (iterable of str, optional) list of domains the parameter is valid over, defaults to empty list

#### new\_copy()

See pybamm. Symbol.new\_copy().

```
class pybamm.FunctionParameter (name, inputs, diff variable=None)
```

A node in the expression tree representing a function parameter

This node will be replaced by a pybamm. Function node if a callable function is passed to the parameter values, and otherwise (in some rarer cases, such as constant current) a pybamm. Scalar node.

#### **Parameters**

- name (str) name of the node
- inputs (dict) A dictionary with string keys and pybamm. Symbol values representing the function inputs. The string keys should provide a reasonable description of what the input to the function is (e.g. "Electrolyte concentration [mol.m-3]")
- diff\_variable (pybamm. Symbol, optional) if diff\_variable is specified, the FunctionParameter node will be replaced by a pybamm. Function and then differentiated with respect to diff\_variable. Default is None.

```
diff(variable)
```

See pybamm. Symbol. diff().

#### get children domains (children list)

Obtains the unique domain of the children. If the children have different domains then raise an error

```
new_copy()
```

See pybamm. Symbol.new\_copy().

```
set id()
```

See pybamm. Symbol. set id()

#### 3.1.3 Variable

class pybamm. Variable (name, domain=None, auxiliary\_domains=None, bounds=None)

A node in the expression tree represending a dependent variable

This node will be discretised by Discretisation and converted to a pybamm. StateVector node.

#### **Parameters**

- name (str) name of the node domain: iterable of str, optional list of domains that this variable is valid over
- auxiliary\_domains (dict, optional) dictionary of auxiliary domains ({'secondary': ..., 'tertiary': ...}). For example, for the single particle model, the particle concentration would be a Variable with domain 'negative particle' and secondary auxiliary domain 'current collector'. For the DFN, the particle concentration would be a Variable with domain 'negative particle', secondary domain 'negative electrode' and tertiary domain 'current collector'
- bounds (tuple, optional) Physical bounds on the variable
- \*Extends -

#### diff(variable)

Differentiate a symbol with respect to a variable. For any symbol that can be differentiated, return 1 if differentiating with respect to yourself, self.\_diff(variable) if variable is in the expression tree of the symbol, and zero otherwise.

Parameters variable (pybamm. Symbol) - The variable with respect to which to differentiate

**class** pybamm. **VariableDot** (*name*, *domain=None*, *auxiliary\_domains=None*, *bounds=None*)

A node in the expression tree represending the time derivative of a dependent variable

This node will be discretised by Discretisation and converted to a pybamm. StateVectorDot node.

#### **Parameters**

- name (str) name of the node
- domain (iterable of str) list of domains that this variable is valid over
- auxiliary\_domains (dict) dictionary of auxiliary domains ({'secondary': ..., 'tertiary': ...}). For example, for the single particle model, the particle concentration would be a Variable with domain 'negative particle' and secondary auxiliary domain 'current collector'. For the DFN, the particle concentration would be a Variable with domain 'negative particle', secondary domain 'negative electrode' and tertiary domain 'current collector'
- **bounds** (*tuple*, *optional*) Physical bounds on the variable. Included for compatibility with *VariableBase*, but ignored.
- \*Extends -

#### diff(variable)

Differentiate a symbol with respect to a variable. For any symbol that can be differentiated, return 1 if differentiating with respect to yourself, self.\_diff(variable) if variable is in the expression tree of the symbol, and zero otherwise.

**Parameters variable** (pybamm. Symbol) – The variable with respect to which to differentiate

#### get\_variable()

return a Variable corresponding to this VariableDot

Note: Variable.\_jac adds a dash to the name of the corresponding VariableDot, so we remove this here

class pybamm. ExternalVariable (name, size, domain=None, auxiliary\_domains=None)

A node in the expression tree representing an external variable variable

This node will be discretised by Discretisation and converted to a Vector node.

#### **Parameters**

- name (str) name of the node
- domain (iterable of str) list of domains that this variable is valid over
- auxiliary\_domains (dict) dictionary of auxiliary domains ({'secondary': ..., 'tertiary': ...}). For example, for the single particle model, the particle concentration would be a Variable with domain 'negative particle' and secondary auxiliary domain 'current collector'. For the DFN, the particle concentration would be a Variable with domain 'negative particle', secondary domain 'negative electrode' and tertiary domain 'current collector'
- \*Extends -

#### diff(variable)

Differentiate a symbol with respect to a variable. For any symbol that can be differentiated, return 1 if differentiating with respect to yourself, self.\_diff(variable) if variable is in the expression tree of the symbol, and zero otherwise.

**Parameters variable** (pybamm. Symbol) – The variable with respect to which to differentiate

#### size

Size of an object, found by evaluating it with appropriate t and y

### 3.1.4 Independent Variable

class pybamm.IndependentVariable (name, domain=None, auxiliary\_domains=None)

A node in the expression tree representing an independent variable

Used for expressing functions depending on a spatial variable or time

#### **Parameters**

- name (str) name of the node
- domain (iterable of str) list of domains that this variable is valid over
- \*Extends -

#### class pybamm.Time

A node in the expression tree representing time

```
Extends: Symbol
new_copy()
```

See pybamm.Symbol.new\_copy().

**class** pybamm. **SpatialVariable** (name, domain=None, auxiliary\_domains=None, coord\_sys=None)

A node in the expression tree representing a spatial variable

#### **Parameters**

- name (str) name of the node (e.g. "x", "y", "z", "r", "x\_n", "x\_s", "x\_p", "r\_n", "r\_p")
- **domain** (*iterable of str*) list of domains that this variable is valid over (e.g. "cartesian", "spherical polar")
- \*Extends -

```
new_copy()
    See pybamm.Symbol.new_copy().
```

### pybamm.t = the independent variable time

A node in the expression tree representing time

Extends: Symbol

### 3.1.5 Scalar

```
class pybamm.Scalar(value, name=None, domain=[])
```

A node in the expression tree representing a scalar value

Extends: Symbol

#### **Parameters**

- ullet value (numeric) the value returned by the node when evaluated
- name (str, optional) the name of the node. Defaulted to str(value) if not provided
- domain (iterable of str, optional) list of domains the parameter is valid over, defaults to empty list

```
new copy()
```

See pybamm.Symbol.new\_copy().

```
set_id()
    See pybamm.Symbol.set_id().
value
```

the value returned by the node when evaluated

## 3.1.6 **Array**

#### **Parameters**

- **entries** (numpy.array or list) the array associated with the node. If a list is provided, it is converted to a numpy array
- name (str, optional) the name of the node
- domain (iterable of str, optional) list of domains the parameter is valid over, defaults to empty list
- auxiliary\_domainds (dict, optional) dictionary of auxiliary domains, defaults to empty dict
- **entries\_string** (*str*) String representing the entries (slow to recalculate when copying)
- \*Extends -

#### ndim

returns the number of dimensions of the tensor

```
new_copy()
    See pybamm.Symbol.new_copy().
set_id()
    See pybamm.Symbol.set_id().
```

#### shape

returns the number of entries along each dimension

```
pybamm.linspace(start, stop, num=50, **kwargs)
```

Creates a linearly spaced array by calling *numpy.linspace* with keyword arguments 'kwargs'. For a list of 'kwargs' see the numpy linspace documentation

```
pybamm.meshgrid(x, y, **kwargs)
```

Return coordinate matrices as from coordinate vectors by calling *numpy.meshgrid* with keyword arguments 'kwargs'. For a list of 'kwargs' see the numpy meshgrid documentation

#### 3.1.7 Matrix

### 3.1.8 Vector

**Extends:** Array

### 3.1.9 State Vector

**class** pybamm. **StateVector** (\*y\_slices, name=None, domain=None, auxiliary\_domains=None, evaluation\_array=None)

node in the expression tree that holds a slice to read from an external vector type

#### **Parameters**

- y\_slice (slice) the slice of an external y to read
- name (str, optional) the name of the node
- domain (iterable of str, optional) list of domains the parameter is valid over, defaults to empty list
- auxiliary\_domains (dict of str, optional) dictionary of auxiliary domains
- **evaluation\_array** (*list*, *optional*) List of boolean arrays representing slices. Default is None, in which case the evaluation\_array is computed from y\_slices.
- \*Extends -

#### diff(variable)

Differentiate a symbol with respect to a variable. For any symbol that can be differentiated, return 1 if differentiating with respect to yourself, self.\_diff(variable) if variable is in the expression tree of the symbol, and zero otherwise.

**Parameters variable** (pybamm. Symbol) – The variable with respect to which to differentiate

#### Parameters

- **y\_slice** (*slice*) the slice of an external ydot to read
- name (str, optional) the name of the node
- domain (iterable of str, optional) list of domains the parameter is valid over, defaults to empty list
- auxiliary\_domains (dict of str, optional) dictionary of auxiliary domains
- **evaluation\_array** (*list*, *optional*) List of boolean arrays representing slices. Default is None, in which case the evaluation\_array is computed from y\_slices.
- \*Extends -

#### diff(variable)

Differentiate a symbol with respect to a variable. For any symbol that can be differentiated, return I

if differentiating with respect to yourself, *self.\_diff(variable)* if *variable* is in the expression tree of the symbol, and zero otherwise.

**Parameters variable** (pybamm. Symbol) – The variable with respect to which to differentiate

## 3.1.10 Binary Operators

```
class pybamm.BinaryOperator(name, left, right)
     A node in the expression tree representing a binary operator (e.g. +, *)
     Derived classes will specify the particular operator
     Extends: Symbol
          Parameters
                • name (str) - name of the node
                • left (Symbol or Number) - lhs child node (converted to Scalar if Number)
                • right (Symbol or Number) - rhs child node (converted to Scalar if Number)
     evaluate (t=None, y=None, y dot=None, inputs=None, known evals=None)
          See pybamm. Symbol. evaluate().
     evaluates_on_edges (dimension)
          See pybamm. Symbol. evaluates on edges ().
     format (left, right)
          Format children left and right into compatible form
     get_children_domains (ldomain, rdomain)
          Combine domains from children in appropriate way
     new_copy()
          See pybamm. Symbol.new_copy().
class pybamm.Power(left, right)
     A node in the expression tree representing a ** power operator
     Extends: BinaryOperator
class pybamm.Addition(left, right)
     A node in the expression tree representing an addition operator
     Extends: BinaryOperator
class pybamm.Subtraction(left, right)
     A node in the expression tree representing a subtraction operator
     Extends: BinaryOperator
class pybamm.Multiplication (left, right)
     A node in the expression tree representing a multiplication operator (Hadamard product). Overloads cases where
     the "*" operator would usually return a matrix multiplication (e.g. scipy.sparse.coo.coo_matrix)
     Extends: BinaryOperator
class pybamm.MatrixMultiplication(left, right)
     A node in the expression tree representing a matrix multiplication operator
     Extends: BinaryOperator
```

```
diff(variable)
    See pybamm.Symbol.diff().
```

#### class pybamm.Division(left, right)

A node in the expression tree representing a division operator

Extends: BinaryOperator

```
class pybamm.Inner(left, right)
```

A node in the expression tree which represents the inner (or dot) product. This operator should be used to take the inner product of two mathematical vectors (as opposed to the computational vectors arrived at post-discretisation) of the form  $v = v_x e_x + v_y e_y + v_z e_z$  where  $v_x, v_y, v_z$  are scalars and  $e_x, e_y, e_z$  are x-y-z-directional unit vectors. For v and w mathematical vectors, inner product returns  $v_x * w_x + v_y * w_y + v_z * w_z$ . In addition, for some spatial discretisations mathematical vector quantities (such as i = grad(phi)) are evaluated on a different part of the grid to mathematical scalars (e.g. for finite volume mathematical scalars are evaluated on the nodes but mathematical vectors are evaluated on cell edges). Therefore, inner also transfers the inner product of the vector onto the scalar part of the grid if required by a particular discretisation.

Extends: BinaryOperator
evaluates\_on\_edges (dimension)
 See pybamm.Symbol.evaluates on edges().

```
class pybamm.Heaviside(name, left, right)
```

A node in the expression tree representing a heaviside step function.

Adding this operation to the rhs or algebraic equations in a model can often cause a discontinuity in the solution. For the specific cases listed below, this will be automatically handled by the solver. In the general case, you can explicitly tell the solver of discontinuities by adding a *Event* object with *EventType* DISCONTINUITY to the model's list of events.

In the case where the Heaviside function is of the form pybamm.t < x, pybamm.t <= x, x < pybamm.t, or x <= pybamm.t, where x is any constant equation, this DISCONTINUITY event will automatically be added by the solver.

```
Extends: BinaryOperator
diff(variable)
    See pybamm.Symbol.diff().
```

#### class pybamm.EqualHeaviside(left, right)

A heaviside function with equality (return 1 when left = right)

class pybamm.NotEqualHeaviside(left, right)

A heaviside function without equality (return 0 when left = right)

**class** pybamm.**Minimum** (*left*, *right*)

Returns the smaller of two objects

class pybamm.Maximum(left, right)

Returns the smaller of two objects

pybamm.minimum(left, right)

Returns the smaller of two objects. Not to be confused with pybamm.min(), which returns min function of child.

```
pybamm.maximum(left, right)
```

Returns the larger of two objects. Not to be confused with pybamm.max(), which returns max function of child.

```
pybamm.source(left, right, boundary=False)
```

A convinience function for creating (part of) an expression tree representing a source term. This is necessary for

spatial methods where the mass matrix is not the identity (e.g. finite element formulation with piecwise linear basis functions). The left child is the symbol representing the source term and the right child is the symbol of the equation variable (currently, the finite element formulation in PyBaMM assumes all functions are constructed using the same basis, and the matrix here is constructed accounting for the boundary conditions of the right child). The method returns the matrix-vector product of the mass matrix (adjusted to account for any Dirichlet boundary conditions imposed the the right symbol) and the discretised left symbol.

#### **Parameters**

- **left** (Symbol) The left child node, which represents the expression for the source term.
- **right** (*Symbol*) The right child node. This is the symbol whose boundary conditions are accounted for in the construction of the mass matrix.
- **boundary** (bool, optional) If True, then the mass matrix should is assembled over the boundary, corresponding to a source term which only acts on the boundary of the domain. If False (default), the matrix is assembled over the entire domain, corresponding to a source term in the bulk.

## 3.1.11 Unary Operators

```
class pybamm. UnaryOperator (name, child, domain=None, auxiliary domains=None)
     A node in the expression tree representing a unary operator (e.g. '-', grad, div)
     Derived classes will specify the particular operator
     Extends: Symbol
          Parameters
                • name (str) – name of the node
                • child (Symbol) - child node
     evaluate (t=None, y=None, y_dot=None, inputs=None, known_evals=None)
          See pybamm. Symbol. evaluate().
     evaluates_on_edges (dimension)
          See pybamm. Symbol. evaluates_on_edges().
     new_copy()
          See pybamm. Symbol. new copy ().
class pybamm.Negate(child)
     A node in the expression tree representing a - negation operator
     Extends: UnaryOperator
class pybamm.AbsoluteValue(child)
     A node in the expression tree representing an abs operator
     Extends: UnaryOperator
     diff(variable)
          See pybamm. Symbol. diff().
class pybamm.Sign(child)
     A node in the expression tree representing a sign operator
     Extends: UnaryOperator
     diff(variable)
          See pybamm. Symbol. diff().
```

```
class pybamm. Index (child, index, name=None, check size=True)
```

A node in the expression tree, which stores the index that should be extracted from its child after the child has been evaluated.

#### **Parameters**

- child (pybamm. Symbol) The symbol of which to take the index
- index (int or slice) The index (if int) or indices (if slice) to extract from the symbol
- name (str, optional) The name of the symbol
- **check\_size** (bool, optional) Whether to check if the slice size exceeds the child size. Default is True. This should always be True when creating a new symbol so that the appropriate check is performed, but should be False for creating a new copy to avoid unnecessarily repeating the check.

```
evaluates_on_edges (dimension)
```

```
See pybamm.Symbol.evaluates_on_edges().
```

```
set id()
```

See pybamm. Symbol. set\_id()

class pybamm. SpatialOperator (name, child, domain=None, auxiliary\_domains=None)

A node in the expression tree representing a unary spatial operator (e.g. grad, div)

Derived classes will specify the particular operator

This type of node will be replaced by the Discretisation class with a Matrix

**Extends:** *UnaryOperator* 

#### **Parameters**

- name (str) name of the node
- child (Symbol) child node

```
diff(variable)
```

See pybamm. Symbol. diff().

#### class pybamm.Gradient(child)

A node in the expression tree representing a grad operator

```
Extends: SpatialOperator
```

```
evaluates_on_edges (dimension)
```

```
See pybamm.Symbol.evaluates_on_edges().
```

class pybamm.Divergence(child)

A node in the expression tree representing a div operator

```
Extends: SpatialOperator
```

```
evaluates_on_edges (dimension)
```

```
See pybamm.Symbol.evaluates_on_edges().
```

```
class pybamm.Laplacian(child)
```

A node in the expression tree representing a laplacian operator. This is currently only implemeted in the weak form for finite element formulations.

```
Extends: SpatialOperator
```

```
evaluates_on_edges (dimension)
```

```
See pybamm. Symbol. evaluates on edges ().
```

#### class pybamm.Gradient\_Squared(child)

A node in the expression tree representing a the inner product of the grad operator with itself. In particular, this is useful in the finite element formulation where we only require the (sclar valued) square of the gradient, and not the gradient itself. **Extends:** SpatialOperator

#### evaluates\_on\_edges (dimension)

See pybamm.Symbol.evaluates\_on\_edges().

#### class pybamm.Mass(child)

Returns the mass matrix for a given symbol, accounting for Dirchlet boundary conditions where necessary (e.g. in the finite element formulation) **Extends:** SpatialOperator

#### class pybamm.Integral (child, integration\_variable)

A node in the expression tree representing an integral operator

$$I = \int_{a}^{b} f(u) \, du,$$

where a and b are the left-hand and right-hand boundaries of the domain respectively, and  $u \in$  domain.

#### **Parameters**

- **function** (*pybamm.Symbol*) The function to be integrated (will become self.children[0])
- integration\_variable (pybamm. Independent Variable) The variable over which to integrate
- \*\*Extends (\*\* SpatialOperator) -

#### evaluates\_on\_edges (dimension)

See pybamm.Symbol.evaluates\_on\_edges().

## $\mathtt{set\_id}()$

See pybamm.Symbol.set\_id()

#### class pybamm.IndefiniteIntegral (child, integration\_variable)

A node in the expression tree representing an indefinite integral operator

$$I = \int_{x_e x t min}^{x} f(u) \, du$$

where  $u \in \text{domain}$  which can represent either a spatial or temporal variable.

#### **Parameters**

- **function** (pybamm.Symbol) The function to be integrated (will become self.children[0])
- integration\_variable (pybamm.IndependentVariable) The variable over which to integrate
- \*\*Extends (\*\* BaseIndefiniteIntegral) -

#### class pybamm.DefiniteIntegralVector(child, vector\_type='row')

A node in the expression tree representing an integral of the basis used for discretisation

$$I = \int_{a}^{b} \psi(x) \, dx,$$

where a and b are the left-hand and right-hand boundaries of the domain respectively and  $\psi$  is the basis function.

#### **Parameters**

- variable (pybamm.Symbol) The variable whose basis will be integrated over the entire domain
- **vector\_type** (*str*, *optional*) Whether to return a row or column vector (default is row)
- \*\*Extends (\*\* SpatialOperator) -

#### set id()

See pybamm. Symbol. set id()

#### class pybamm.BoundaryIntegral (child, region='entire')

A node in the expression tree representing an integral operator over the boundary of a domain

$$I = \int_{\partial a} f(u) \, du,$$

where  $\partial a$  is the boundary of the domain, and  $u \in$  domain boundary.

#### **Parameters**

- **function** (*pybamm.Symbol*) The function to be integrated (will become self.children[0])
- **region** (*str*, *optional*) The region of the boundary over which to integrate. If region is *entire* (default) the integration is carried out over the entire boundary. If region is *negative tab* or *positive tab* then the integration is only carried out over the appropriate part of the boundary corresponding to the tab.
- \*\*Extends (\*\* SpatialOperator) -

#### evaluates\_on\_edges (dimension)

See pybamm. Symbol. evaluates\_on\_edges().

set\_id()

See pybamm.Symbol.set\_id()

#### class pybamm.DeltaFunction(child, side, domain)

Delta function. Currently can only be implemented at the edge of a domain

#### **Parameters**

- child (pybamm. Symbol) The variable that sets the strength of the delta function
- **side** (str) Which side of the domain to implement the delta function on
- \*\*Extends (\*\* SpatialOperator) -

#### evaluate\_for\_shape()

See pybamm.Symbol.evaluate\_for\_shape\_using\_domain()

#### evaluates\_on\_edges (dimension)

See pybamm.Symbol.evaluates\_on\_edges().

set\_id()

See pybamm.Symbol.set\_id()

## class pybamm.BoundaryOperator(name, child, side)

A node in the expression tree which gets the boundary value of a variable.

#### **Parameters**

- name (str) The name of the symbol
- child (pybamm. Symbol) The variable whose boundary value to take

```
• side (str) – Which side to take the boundary value on ("left" or "right")
                • **Extends (** SpatialOperator) -
     set_id()
          See pybamm.Symbol.set_id()
class pybamm.BoundaryValue(child, side)
     A node in the expression tree which gets the boundary value of a variable.
          Parameters
                • child (pybamm. Symbol) - The variable whose boundary value to take
                • side (str) – Which side to take the boundary value on ("left" or "right")
                • **Extends (** BoundaryOperator) -
class pybamm.BoundaryGradient(child, side)
     A node in the expression tree which gets the boundary flux of a variable.
          Parameters
                • child (pybamm. Symbol) - The variable whose boundary flux to take
                • side (str) – Which side to take the boundary flux on ("left" or "right")
                • **Extends (** BoundaryOperator) -
pybamm.grad(expression)
     convenience function for creating a Gradient
          Parameters expression (Symbol) – the gradient will be performed on this sub-expression
          Returns the gradient of expression
          Return type Gradient
pybamm.div (expression)
     convenience function for creating a Divergence
          Parameters expression (Symbol) – the divergence will be performed on this sub-expression
          Returns the divergence of expression
          Return type Divergence
pybamm.laplacian (expression)
     convenience function for creating a Laplacian
          Parameters expression (Symbol) – the laplacian will be performed on this sub-expression
          Returns the laplacian of expression
          Return type Laplacian
pybamm.grad_squared(expression)
     convenience function for creating a Gradient_Squared
          Parameters expression (Symbol) – the inner product of the gradient with itself will be per-
              formed on this sub-expression
          Returns inner product of the gradient of expression with itself
          Return type Gradient_Squared
pybamm.surf(symbol)
     convenience function for creating a right BoundaryValue, usually in the spherical geometry
```

```
Parameters symbol (pybamm. Symbol) – the surface value of this symbol will be returned
          Returns the surface value of symbol
          Return type pybamm. Boundary Value
pybamm.x_average(symbol)
     convenience function for creating an average in the x-direction
          Parameters symbol (pybamm. Symbol) – The function to be averaged
          Returns the new averaged symbol
          Return type Symbol
pybamm.r_average(symbol)
     convenience function for creating an average in the r-direction
          Parameters symbol (pybamm. Symbol) - The function to be averaged
          Returns the new averaged symbol
          Return type Symbol
pybamm.z_average(symbol)
     convenience function for creating an average in the z-direction
          Parameters symbol (pybamm. Symbol) - The function to be averaged
          Returns the new averaged symbol
          Return type Symbol
pybamm.yz_average(symbol)
     convenience function for creating an average in the y-z-direction
          Parameters symbol (pybamm. Symbol) – The function to be averaged
          Returns the new averaged symbol
          Return type Symbol
pybamm.boundary_value(symbol, side)
     convenience function for creating a pybamm. Boundary Value
          Parameters
                • symbol (pybamm.Symbol) – The symbol whose boundary value to take
                • side (str) – Which side to take the boundary value on ("left" or "right")
          Returns the new integrated expression tree
          Return type Boundary Value
pybamm.sign(symbol)
     Returns a Sign object.
3.1.12 Concatenations
```

class pybamm.Concatenation(\*children, name=None, check\_domain=True, concat\_fun=None) A node in the expression tree representing a concatenation of symbols

Extends: pybamm.Symbol

Parameters children (iterable of pybamm. Symbol) - The symbols to concatenate

```
evaluate (t=None, y=None, y_dot=None, inputs=None, known_evals=None)
    See pybamm.Symbol.evaluate().
new_copy()
    See pybamm.Symbol.new_copy().
```

#### class pybamm.NumpyConcatenation(\*children)

A node in the expression tree representing a concatenation of equations, when we *don't* care about domains. The class <code>pybamm.DomainConcatenation</code>, which *is* careful about domains and uses broadcasting where appropriate, should be used whenever possible instead.

Upon evaluation, equations are concatenated using numpy concatenation.

Extends: Concatenation

Parameters children (iterable of pybamm. Symbol) - The equations to concatenate

class pybamm.DomainConcatenation(children, full\_mesh, copy\_this=None)

A node in the expression tree representing a concatenation of symbols, being careful about domains.

It is assumed that each child has a domain, and the final concatenated vector will respect the sizes and ordering of domains established in mesh keys

Extends: pybamm.Concatenation

#### **Parameters**

- children (iterable of pybamm. Symbol) The symbols to concatenate
- **full\_mesh** (pybamm.BaseMesh) The underlying mesh for discretisation, used to obtain the number of mesh points in each domain.
- **copy\_this** (*pybamm.DomainConcatenation* (optional)) if provided, this class is initialised by copying everything except the children from *copy\_this. mesh* is not used in this case

### class pybamm.SparseStack(\*children)

A node in the expression tree representing a concatenation of sparse matrices. As with NumpyConcatenation, we *don't* care about domains. The class *pybamm.DomainConcatenation*, which *is* careful about domains and uses broadcasting where appropriate, should be used whenever possible instead.

Extends: Concatenation

Parameters children (iterable of Concatenation) – The equations to concatenate

# 3.1.13 Broadcasting Operators

A node in the expression tree representing a broadcasting operator. Broadcasts a child to a specified domain. After discretisation, this will evaluate to an array of the right shape for the specified domain.

For an example of broadcasts in action, see this example notebook

#### **Parameters**

- child (Symbol) child node
- **broadcast\_domain** (*iterable of str*) Primary domain for broadcast. This will become the domain of the symbol
- broadcast\_auxiliary\_domains (dict of str) Auxiliary domains for broadcast.

- **broadcast\_type** (*str*, *optional*) Whether to broadcast to the full domain (primary and secondary) or only in the primary direction. Default is "full".
- name (str) name of the node
- \*\*Extends (\*\* SpatialOperator) -

class pybamm.FullBroadcast (child, broadcast\_domain, auxiliary\_domains, name=None)

A class for full broadcasts

class pybamm.PrimaryBroadcast (child, broadcast\_domain, name=None)

A node in the expression tree representing a primary broadcasting operator. Broadcasts in a *primary* dimension only. That is, makes explicit copies of the symbol in the domain specified by  $broadcast\_domain$ . This should be used for broadcasting from a "larger" scale to a "smaller" scale, for example broadcasting temperature T(x) from the electrode to the particles, or broadcasting current collector current i(y, z) from the current collector to the electrodes.

#### **Parameters**

- child (Symbol) child node
- **broadcast\_domain** (*iterable of str*) Primary domain for broadcast. This will become the domain of the symbol
- name (str) name of the node
- \*\*Extends (\*\* SpatialOperator) -

class pybamm.SecondaryBroadcast (child, broadcast\_domain, name=None)

A node in the expression tree representing a primary broadcasting operator. Broadcasts in a *secondary* dimension only. That is, makes explicit copies of the symbol in the domain specified by *broadcast\_domain*. This should be used for broadcasting from a "smaller" scale to a "larger" scale, for example broadcasting SPM particle concentrations c\_s(r) from the particles to the electrodes. Note that this wouldn't be used to broadcast particle concentrations in the DFN, since these already depend on both x and r.

#### **Parameters**

- child (Symbol) child node
- **broadcast\_domain** (*iterable of str*) Primary domain for broadcast. This will become the domain of the symbol
- name (str) name of the node
- \*\*Extends (\*\* SpatialOperator) -

class pybamm.FullBroadcastToEdges (child, broadcast\_domain, auxiliary\_domains, name=None)
A full broadcast onto the edges of a domain (edges of primary dimension, nodes of other dimensions)

```
evaluates_on_edges (dimension)
See pybamm.Symbol.evaluates_on_edges().
```

```
class pybamm.PrimaryBroadcastToEdges (child, broadcast_domain, name=None)
     A primary broadcast onto the edges of the domain
     evaluates_on_edges (dimension)
         See pybamm. Symbol. evaluates_on_edges().
class pybamm. SecondaryBroadcastToEdges (child, broadcast domain, name=None)
     A secondary broadcast onto the edges of a domain
     evaluates on edges (dimension)
         See pybamm. Symbol. evaluates_on_edges().
pybamm.ones_like(*symbols)
     Create a symbol with the same shape as the input symbol and with constant value '1', using FullBroadcast.
         Parameters symbols (Symbol) – Symbols whose shape to copy
3.1.14 Functions
```

```
class pybamm.Function(function,
                                        *children,
                                                                   derivative='autograd',
                                                                                           differenti-
                                                   name=None,
                             ated function=None)
     A node in the expression tree representing an arbitrary function
```

# **Parameters**

- function (method) A function can have 0 or many inputs. If no inputs are given, self.evaluate() simply returns func(). Otherwise, self.evaluate(t, y, u) returns func(child0.evaluate(t, y, u), child1.evaluate(t, y, u), etc).
- children (pybamm. Symbol) The children nodes to apply the function to
- **derivative** (str, optional) Which derivative to use when differentiating ("autograd" or "derivative"). Default is "autograd".
- differentiated\_function (method, optional) The function which was differentiated to obtain this one. Default is None.

```
• **Extends (** pybamm.Symbol) -
diff(variable)
    See pybamm. Symbol. diff().
evaluate (t=None, y=None, y_dot=None, inputs=None, known_evals=None)
    See pybamm. Symbol. evaluate().
evaluates on edges (dimension)
    See pybamm. Symbol. evaluates_on_edges().
get_children_domains (children_list)
    Obtains the unique domain of the children. If the children have different domains then raise an error
new_copy()
    See pybamm. Symbol.new_copy().
```

# class pybamm.SpecificFunction (function, child)

Parent class for the specific functions, which implement their own diff operators directly.

### **Parameters**

- **function** (method) Function to be applied to child
- **child** (pybamm. Symbol) The child to apply the function to

```
class pybamm.Cos(child)
     Cosine function
pybamm.cos(child)
     Returns cosine function of child.
class pybamm.Cosh(child)
     Hyberbolic cosine function
pybamm.cosh(child)
     Returns hyperbolic cosine function of child.
class pybamm.Exponential(child)
     Exponential function
pybamm.exp(child)
     Returns exponential function of child.
class pybamm.Log(child)
     Logarithmic function
pybamm.log(child, base='e')
     Returns logarithmic function of child (any base, default 'e').
pybamm.max(child)
     Returns max function of child. Not to be confused with pybamm.maximum(), which returns the larger of two
     objects.
pybamm.min(child)
     Returns min function of child. Not to be confused with pybamm.minimum(), which returns the smaller of
     two objects.
class pybamm.Sin(child)
     Sine function
pybamm.sin(child)
     Returns sine function of child.
class pybamm.Sinh(child)
     Hyperbolic sine function
pybamm.sinh(child)
     Returns hyperbolic sine function of child.
3.1.15 Input Parameter
class pybamm.InputParameter(name, domain=None)
     A node in the expression tree representing an input parameter
     This node's value can be set at the point of solving, allowing parameter estimation and control
          Parameters
                • name (str) – name of the node
                • domain (iterable of str, or str) - list of domains over which the node is valid
                  (empty list indicates the symbol is valid over all domains)
     new_copy()
```

See pybamm.Symbol.new\_copy().

Specify the size that the input parameter should be

set expected size (size)

# 3.1.16 Interpolant

### **Parameters**

- data (numpy.ndarray) Numpy array of data to use for interpolation. Must have exactly two columns (x and y data)
- child (pybamm. Symbol) Node to use when evaluating the interpolant
- name (str, optional) Name of the interpolant. Default is None, in which case the name "interpolating function" is given.
- interpolator (str, optional) Which interpolator to use ("pchip" or "cubic spline"). Note that whichever interpolator is used must be differentiable (for Interpolator.\_diff). Default is "cubic spline". Note that "pchip" may give slow results.
- **extrapolate** (bool, optional) Whether to extrapolate for points that are outside of the parametrisation range, or return NaN (following default behaviour from scipy). Default is True.
- \*\*Extends\*\* (pybamm.Function) -

```
set_id()
```

See pybamm.Symbol.set\_id().

# 3.1.17 Operations on expression trees

Classes and functions that operate on the expression tree

### **Simplify**

class pybamm.Simplification(simplified\_symbols=None)

```
simplify (symbol, clear_domains=True)
```

This function recurses down the tree, applying any simplifications defined in classes derived from py-bamm.Symbol. E.g. any expression multiplied by a pybamm.Scalar(0) will be simplified to a py-bamm.Scalar(0). If a symbol has already been simplified, the stored value is returned.

### **Parameters**

- symbol (pybamm. Symbol) The symbol to simplify
- clear\_domains (bool) Whether to remove a symbol's domain when simplifying. Default is True.

#### Returns

- pybamm.Symbol
- Simplified symbol

pybamm.simplify\_if\_constant(symbol, keep\_domains=False)

Utility function to simplify an expression tree if it evalutes to a constant scalar, vector or matrix

#### pybamm.simplify\_addition\_subtraction(myclass, left, right)

if children are associative (addition, subtraction, etc) then try to find groups of constant children (that produce a value) and simplify them to a single term

The purpose of this function is to simplify expressions like (1 + (1 + p)), which should be simplified to (2 + p). The former expression consists of an Addition, with a left child of Scalar type, and a right child of another Addition containing a Scalar and a Parameter. For this case, this function will first flatten the expression to a list of the bottom level children (i.e. [Scalar(1), Scalar(2), Parameter(p)]), and their operators (i.e. [None, Addition, Addition]), and then combine all the constant children (i.e. Scalar(1) and Scalar(1)) to a single child (i.e. Scalar(2))

Note that this function will flatten the expression tree until a symbol is found that is not either an Addition or a Subtraction, so this function would simplify (3 - (2 + a\*b\*c)) to (1 + a\*b\*c)

This function is useful if different children expressions contain non-constant terms that prevent them from being simplified, so for example (1 + a) + (b - 2) - (6 + c) will be simplified to (-7 + a + b - c)

#### **Parameters**

- myclass (class) the binary operator class (pybamm.Addition or pybamm.Subtraction) operating on children left and right
- left (derived from pybamm. Symbol) the left child of the binary operator
- right (derived from pybamm. Symbol) the right child of the binary operator

### pybamm.simplify\_multiplication\_division (myclass, left, right)

if children are associative (multiply, division, etc) then try to find groups of constant children (that produce a value) and simplify them

The purpose of this function is to simplify expressions of the type (1 \* c / 2), which should simplify to (0.5 \* c). The former expression consists of a Division, with a left child of a Multiplication containing a Scalar and a Parameter, and a right child consisting of a Scalar. For this case, this function will first flatten the expression to a list of the bottom level children on the numerator (i.e. [Scalar(1), Parameter(c)]) and their operators (i.e. [None, Multiplication]), as well as those children on the denominator (i.e. [Scalar(2)]. After this, all the constant children on the numerator and denominator (i.e. Scalar(1) and Scalar(2)) will be combined appropriately, in this case to Scalar(0.5), and combined with the nonconstant children (i.e. Parameter(c))

Note that this function will flatten the expression tree until a symbol is found that is not either an Multiplication, Division or MatrixMultiplication, so this function would simplify (3\*(1+d)\*2) to (6\*(1+d))

As well as Multiplication and Division, this function can handle MatrixMultiplication. If any MatrixMultiplications are found on the numerator/denominator, no reordering of children is done to find groups of constant children. In this case only neighbouring constant children on the numerator are simplified

#### **Parameters**

- **myclass** (*class*) the binary operator class (pybamm.Addition or pybamm.Subtraction) operating on children left and right
- left (derived from pybamm. Symbol) the left child of the binary operator
- right (derived from pybamm.Symbol) the right child of the binary operator

# **EvaluatorPython**

### class pybamm.EvaluatorPython(symbol)

Converts a pybamm expression tree into pure python code that will calculate the result of calling *evaluate(t, y)* on the given expression tree.

**Parameters** symbol (pybamm. Symbol) – The symbol to convert to python code

**evaluate** (*t*=*None*, *y*=*None*, *y*\_*dot*=*None*, *inputs*=*None*, *known*\_*evals*=*None*) Acts as a drop-in replacement for *pybamm*. *Symbol*. *evaluate*()

### Jacobian

**class** pybamm. **Jacobian** (*known\_jacs=None*, *clear\_domain=True*) Helper class to calculate the jacobian of an expression.

#### **Parameters**

- known\_jacs (dict {variable ids -> pybamm.Symbol}) cached jacobians
- clear\_domain (bool) wether or not the jacobian clears the domain (default True)

jac (symbol, variable)

This function recurses down the tree, computing the Jacobian using the Jacobians defined in classes derived from pybamm. Symbol. E.g. the Jacobian of a 'pybamm. Multiplication' is computed via the product rule. If the Jacobian of a symbol has already been calculated, the stored value is returned. Note: The Jacobian is the derivative of a symbol with respect to a (slice of) a State Vector.

#### **Parameters**

- symbol (pybamm.Symbol) The symbol to calculate the Jacobian of
- variable (pybamm. Symbol) The variable with respect to which to differentiate

Returns Symbol representing the Jacobian

Return type pybamm. Symbol

#### Convert to CasADi

class pybamm.CasadiConverter(casadi\_symbols=None)

convert (symbol, t, y, y\_dot, inputs)

This function recurses down the tree, converting the PyBaMM expression tree to a CasADi expression tree

#### **Parameters**

- symbol (pybamm. Symbol) The symbol to convert
- t (casadi.MX) A casadi symbol representing time
- y (casadi.MX) A casadi symbol representing state vectors
- y\_dot (casadi.MX) A casadi symbol representing time derivatives of state vectors
- inputs (dict) A dictionary of casadi symbols representing parameters

**Returns** The converted symbol

Return type casadi.MX

#### **Symbol Unpacker**

class pybamm.SymbolUnpacker(classes\_to\_find, unpacked\_symbols=None)

Helper class to unpack a (set of) symbol(s) to find all instances of a class. Uses caching to speed up the process.

#### **Parameters**

- classes\_to\_find(list of pybamm classes) Classes to identify in the equations
- unpacked\_symbols (dict {variable ids -> pybamm.Symbol}) cached unpacked equations

# unpack\_list\_of\_symbols(list\_of\_symbols)

Unpack a list of symbols. See SymbolUnpacker.unpack()

Parameters list\_of\_symbols (list of pybamm. Symbol) - List of symbols to unpack

**Returns** List of unpacked symbols with class in *self.classes\_to\_find* 

Return type list of pybamm. Symbol

### unpack\_symbol (symbol)

This function recurses down the tree, unpacking the symbols and saving the ones that have a class in self.classes\_to\_find.

Parameters symbol (list of pybamm. Symbol) - The symbols to unpack

**Returns** List of unpacked symbols with class in self.classes\_to\_find

Return type list of pybamm. Symbol

# 3.2 Models

Below is an overview of all the battery models included in PyBaMM. Each of the pre-built models contains a reference to the paper in which it is derived.

The models can be customised using the *options* dictionary defined in the *pybamm.BaseBatteryModel* (which also provides information on which options and models are compatible) Visit our examples page to see how these models can be solved, and compared, using PyBaMM.

## 3.2.1 Base Models

#### **Base Model**

class pybamm.BaseModel(name='Unnamed model')

Base model class for other models to extend.

#### name

A string giving the name of the model

Type str

## options

A dictionary of options to be passed to the model

Type dict

#### rhs

A dictionary that maps expressions (variables) to expressions that represent the rhs

Type dict

### algebraic

A dictionary that maps expressions (variables) to expressions that represent the algebraic equations. The algebraic expressions are assumed to equate to zero. Note that all the variables in the model must exist in the keys of *rhs* or *algebraic*.

### Type dict

#### initial conditions

A dictionary that maps expressions (variables) to expressions that represent the initial conditions for the state variables y. The initial conditions for algebraic variables are provided as initial guesses to a root finding algorithm that calculates consistent initial conditions.

```
Type dict
```

#### boundary conditions

A dictionary that maps expressions (variables) to expressions that represent the boundary conditions

Type dict

#### variables

A dictionary that maps strings to expressions that represent the useful variables

Type dict

#### events

A list of events. Each event can either cause the solver to terminate (e.g. concentration goes negative), or be used to inform the solver of the existance of a discontinuity (e.g. discontinuity in the input current)

Type list of pybamm. Event

#### concatenated rhs

After discretisation, contains the expressions representing the rhs equations concatenated into a single expression

Type pybamm. Concatenation

### concatenated\_algebraic

After discretisation, contains the expressions representing the algebraic equations concatenated into a single expression

Type pybamm.Concatenation

#### concatenated initial conditions

After discretisation, contains the vector of initial conditions

Type numpy.array

### mass\_matrix

After discretisation, contains the mass matrix for the model. This is computed automatically

Type pybamm.Matrix

### mass\_matrix\_inv

After discretisation, contains the inverse mass matrix for the differential (rhs) part of model. This is computed automatically

Type pybamm. Matrix

### jacobian

Contains the Jacobian for the model. If model.use\_jacobian is True, the Jacobian is computed automatically during solver set up

Type pybamm. Concatenation

### jacobian\_rhs

Contains the Jacobian for the part of the model which contains time derivatives. If model.use\_jacobian is True, the Jacobian is computed automatically during solver set up

Type pybamm. Concatenation

### jacobian\_algebraic

Contains the Jacobian for the algebraic part of the model. This may be used by the solver when calculating consistent initial conditions. If model.use\_jacobian is True, the Jacobian is computed automatically during solver set up

Type pybamm. Concatenation

#### use jacobian

Whether to use the Jacobian when solving the model (default is True)

Type bool

### use\_simplify

Whether to simplify the expression tress representing the rhs and algebraic equations, Jacobain (if using) and events, before solving the model (default is True)

Type bool

### convert\_to\_format

Whether to convert the expression trees representing the rhs and algebraic equations, Jacobain (if using) and events into a different format:

- None: keep PyBaMM expression tree structure.
- "python": convert into pure python code that will calculate the result of calling *evaluate(t, y)* on the given expression treeself.
- "casadi": convert into CasADi expression tree, which then uses CasADi's algorithm to calculate the Jacobian.

Default is "casadi".

Type str

### check\_algebraic\_equations (post\_discretisation)

Check that the algebraic equations are well-posed. Before discretisation, each algebraic equation key must appear in the equation After discretisation, there must be at least one StateVector in each algebraic equation

#### check\_default\_variables\_dictionaries()

Chec that the right variables are provided.

#### check\_ics\_bcs()

Check that the initial and boundary conditions are well-posed.

### check\_no\_repeated\_keys()

Check that no equation keys are repeated

### check\_well\_determined(post\_discretisation)

Check that the model is not under- or over-determined.

#### check\_well\_posedness(post\_discretisation=False)

Check that the model is well-posed by executing the following tests: - Model is not over- or underdetermined, by comparing keys and equations in rhs and algebraic. Overdetermined if more equations than variables, underdetermined if more variables than equations. - There is an initial condition in self.initial\_conditions for each variable/equation pair in self.rhs - There are appropriate boundary conditions in self.boundary\_conditions for each variable/equation pair in self.rhs and self.algebraic

**Parameters post\_discretisation** (boolean) – A flag indicating tests to be skipped after discretisation

### default\_solver

Return default solver based on whether model is ODE model or DAE model

#### info(symbol name)

Provides helpful summary information for a symbol.

Parameters parameter\_name(str)-

### input\_parameters

Returns all the input parameters in the model

#### new\_copy (build=False)

Create an empty copy with identical options, or new options if specified. The 'build' parameter is included for compatibility with subclasses, but unused.

#### parameters

Returns all the parameters in the model

#### timescale

Timescale of model, to be used for non-dimensionalising time when solving

### update (\*submodels)

Update model to add new physics from submodels

Parameters submodel (iterable of pybamm.BaseModel) - The submodels from which to create new model

### **Base Battery Model**

class pybamm.BaseBatteryModel (options=None, name='Unnamed battery model')

Base model class with some default settings and required variables

#### options

A dictionary of options to be passed to the model. The options that can be set are listed below. Note that not all of the options are compatible with each other and with all of the models implemented in PyBaMM.

- "dimensionality" [int, optional] Sets the dimension of the current collector problem. Can be 0 (default), 1 or 2.
- "surface form" [bool or str, optional] Whether to use the surface formulation of the problem. Can be False (default), "differential" or "algebraic".
- "convection" [bool or str, optional] Whether to include the effects of convection in the model. Can be False (default), "differential" or "algebraic". Must be 'False' for lithium-ion models.
- "side reactions" [list, optional] Contains a list of any side reactions to include. Default is []. If this list is not empty (i.e. side reactions are included in the model), then "surface form" cannot be 'False'.
- "interfacial surface area" [str, optional] Sets the model for the interfacial surface area. Can be "constant" (default) or "varying". Not currently implemented in any of the models.
- "current collector" [str, optional] Sets the current collector model to use. Can be "uniform" (default), "potential pair" or "potential pair quite conductive".
- "particle" [str, optional] Sets the submodel to use to describe behaviour within the particle. Can be "Fickian diffusion" (default) or "fast diffusion".
- "thermal" [str, optional] Sets the thermal model to use. Can be "isothermal" (default), "lumped", "x-lumped", or "x-full".
- "external submodels" [list] A list of the submodels that you would like to supply an external variable for instead of solving in PyBaMM. The entries of the lists are strings that correspond to the submodel names in the keys of *self.submodels*.

- "sei" [str] Set the sei submodel to be used. Options are:
  - None: pybamm.sei.NoSEI (no SEI growth)
  - "constant": pybamm.sei.Constant (constant SEI thickness)
  - "reaction limited": pybamm.sei.ReactionLimited
  - "solvent-diffusion limited": pybamm.sei.SolventDiffusionLimited
  - "electron-migration limited": pybamm.sei.ElectronMigrationLimited
  - "interstitial-diffusion limited": pybamm.sei.InterstitialDiffusionLimited
  - "ec reaction limited": pybamm.sei.EcReactionLimited
- "sei film resistance" [str] Set the submodel for additional term in the overpotential due to SEI. The default value is "None" if the "sei" option is "None", and "distributed" otherwise. This is because the "distributed" model is more complex than the model with no additional resistance, which adds unnecessary complexity if there is no SEI in the first place
  - None: no additional resistance

$$\eta_r = \frac{F}{RT} * (\phi_s - \phi_e - U)$$

- "distributed": properly included additional resistance term

$$\eta_r = \frac{F}{RT} * (\phi_s - \phi_e - U - R_{sei} * L_{sei} * j)$$

- "average": constant additional resistance term (approximation to the true model). This model can give sin

$$\eta_r = \frac{F}{RT} * (\phi_s - \phi_e - U - R_{sei} * L_{sei} * \frac{I}{aL})$$

• "sei porosity change" [bool] Whether to include porosity change due to SEI formation (default False)

Type dict

### \*\*Extends

Type \*\* pybamm.BaseModel

new\_copy (build=True)

Create a copy of the model. Overwrites the functionality of pybamm.BaseModel to make sure that the submodels are updated correctly

# process\_parameters\_and\_discretise (symbol, parameter\_values, disc)

Process parameters and discretise a symbol using supplied parameter values and discretisation. Note: care should be taken if using spatial operators on dimensional symbols. Operators in pybamm are written in non-dimensional form, so may need to be scaled by the appropriate length scale. It is recommended to use this method on non-dimensional symbols.

### **Parameters**

- symbol (pybamm. Symbol) Symbol to be processed
- parameter\_values (pybamm.ParameterValues) The parameter values to use during processing
- disc (pybamm.Discretisation) The discrisation to use

**Returns** Processed symbol

```
Return type pybamm. Symbol
```

```
set_external_circuit_submodel()
```

Define how the external circuit defines the boundary conditions for the model, e.g. (not necessarily constant-) current, voltage, etc

```
set_soc_variables()
```

Set variables relating to the state of charge. This function is overriden by the base battery models

### **Event**

class pybamm.Event (name, expression, event\_type=<EventType.TERMINATION: 0>)

Defines an event for use within a pybamm model

#### name

A string giving the name of the event

Type str

#### event\_type

An enum defining the type of event

Type pybamm. Event Type

#### expression

An expression that defines when the event occurs

Type pybamm.Symbol

evaluate (t=None, y=None, y\_dot=None, inputs=None, known\_evals=None)

Acts as a drop-in replacement for pybamm. Symbol.evaluate()

### class pybamm.EventType

Defines the type of event, see pybamm. Event

TERMINATION indicates an event that will terminate the solver, the expression should return 0 when the event is triggered

DISCONTINUITY indicates an expected discontinuity in the solution, the expression should return the time that the discontinuity occurs. The solver will integrate up to the discontinuity and then restart just after the discontinuity.

### 3.2.2 Lithium-ion Models

#### **Base Lithium-ion Model**

**class** pybamm.lithium\_ion.**BaseModel** (*options=None*, *name='Unnamed lithium-ion model'*)

Overwrites default parameters from Base Model with default parameters for lithium-ion models

Extends: pybamm.BaseBatteryModel

# Single Particle Model (SPM)

**class** pybamm.lithium\_ion.**SPM**(options=None, name='Single Particle Model', build=True) Single Particle Model (SPM) of a lithium-ion battery, from 1.

<sup>&</sup>lt;sup>1</sup> SG Marquis, V Sulzer, R Timms, CP Please and SJ Chapman. "An asymptotic derivation of a single particle model with electrolyte". Journal of The Electrochemical Society, 166(15):A3693–A3706, 2019

#### **Parameters**

- **options** (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).

#### References

Extends: pybamm.lithium\_ion.BaseModel

class pybamm.lithium\_ion.BasicSPM(name='Single Particle Model')

Single Particle Model (SPM) model of a lithium-ion battery, from<sup>2</sup>.

This class differs from the <code>pybamm.lithium\_ion.SPM</code> model class in that it shows the whole model in a single class. This comes at the cost of flexibility in combining different physical effects, and in general the main SPM class should be used instead.

**Parameters** name (str, optional) – The name of the model.

#### References

Extends: pybamm.lithium\_ion.BaseModel

new\_copy (build=False)

Create a copy of the model. Overwrites the functionality of <code>pybamm.BaseModel</code> to make sure that the submodels are updated correctly

# **Single Particle Model with Electrolyte (SPMe)**

Single Particle Model with Electrolyte (SPMe) of a lithium-ion battery, from<sup>1</sup>.

### **Parameters**

- options (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).

### References

Extends: pybamm.lithium\_ion.BaseModel

<sup>&</sup>lt;sup>2</sup> SG Marquis, V Sulzer, R Timms, CP Please and SJ Chapman. "An asymptotic derivation of a single particle model with electrolyte". Journal of The Electrochemical Society, 166(15):A3693–A3706, 2019

<sup>&</sup>lt;sup>1</sup> SG Marquis, V Sulzer, R Timms, CP Please and SJ Chapman. "An asymptotic derivation of a single particle model with electrolyte". Journal of The Electrochemical Society, 166(15):A3693–A3706, 2019

### **Doyle-Fuller-Newman (DFN)**

**class** pybamm.lithium\_ion.**DFN** (options=None, name='Doyle-Fuller-Newman model', build=True) Doyle-Fuller-Newman (DFN) model of a lithium-ion battery, from .

#### **Parameters**

- options (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).

### References

Extends: pybamm.lithium\_ion.BaseModel

class pybamm.lithium\_ion.BasicDFN (name='Doyle-Fuller-Newman model')

Doyle-Fuller-Newman (DFN) model of a lithium-ion battery, from<sup>2</sup>.

This class differs from the pybamm.lithium\_ion.DFN model class in that it shows the whole model in a single class. This comes at the cost of flexibility in comparing different physical effects, and in general the main DFN class should be used instead.

**Parameters name** (str, optional) – The name of the model.

#### References

Extends: pybamm.lithium\_ion.BaseModel

new\_copy (build=False)

Create a copy of the model. Overwrites the functionality of pybamm.BaseModel to make sure that the submodels are updated correctly

### 3.2.3 Lead Acid Models

#### **Base Model**

class pybamm.lead\_acid.BaseModel (options=None, name='Unnamed lead-acid model')

Overwrites default parameters from Base Model with default parameters for lead-acid models

Extends: pybamm.BaseBatteryModel

set\_soc\_variables()

Set variables relating to the state of charge.

<sup>&</sup>lt;sup>1</sup> SG Marquis, V Sulzer, R Timms, CP Please and SJ Chapman. "An asymptotic derivation of a single particle model with electrolyte". Journal of The Electrochemical Society, 166(15):A3693–A3706, 2019

<sup>&</sup>lt;sup>2</sup> SG Marquis, V Sulzer, R Timms, CP Please and SJ Chapman. "An asymptotic derivation of a single particle model with electrolyte". Journal of The Electrochemical Society, 166(15):A3693–A3706, 2019

### **Leading-Order Quasi-Static Model**

**class** pybamm.lead\_acid.**LOQS** (options=None, name='LOQS model', build=True)
Leading-Order Quasi-Static model for lead-acid, from 1.

#### **Parameters**

- options (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).

### References

Extends: pybamm.lead\_acid.BaseModel

#### set external circuit submodel()

Define how the external circuit defines the boundary conditions for the model, e.g. (not necessarily constant-) current, voltage, etc

### **Higher-Order Models**

Base model for higher-order models for lead-acid, from 1. Uses leading-order model from pybamm. lead acid.LOQS

### **Parameters**

- **options** (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).

#### References

Extends: pybamm.lead acid.BaseModel

#### set\_full\_convection\_submodel()

Update convection submodel, now that we have the spatially heterogeneous interfacial current densities

### set full interface submodel()

Set full interface submodel, to get spatially heterogeneous interfacial current densities

### set\_full\_porosity\_submodel()

Update porosity submodel, now that we have the spatially heterogeneous interfacial current densities

<sup>&</sup>lt;sup>1</sup> V Sulzer, SJ Chapman, CP Please, DA Howey, and CW Monroe. Faster lead-acid battery simulations from porous-electrode theory: Part II. Asymptotic analysis. Journal of The Electrochemical Society 166.12 (2019), A2372–A2382.

<sup>&</sup>lt;sup>1</sup> V Sulzer, SJ Chapman, CP Please, DA Howey, and CW Monroe. Faster lead-acid battery simulations from porous-electrode theory: Part II. Asymptotic analysis. Journal of The Electrochemical Society 166.12 (2019), A2372–A2382.

class pybamm.lead\_acid.FOQS (options=None, name='FOQS model', build=True)
 First-order quasi-static model for lead-acid, from¹. Uses leading-order model from pybamm.lead\_acid.
 LOOS

### **Parameters**

- options (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).
- \*\*Extends (\*\* pybamm.lead\_acid.BaseHigherOrderModel) -

### set\_full\_porosity\_submodel()

Update porosity submodel, now that we have the spatially heterogeneous interfacial current densities

class pybamm.lead\_acid.Composite (options=None, name='Composite model', build=True)

Composite model for lead-acid, from 1. Uses leading-order model from pybamm.lead\_acid.LOQS

Extends: pybamm.lead\_acid.BaseHigherOrderModel

### set\_full\_porosity\_submodel()

Update porosity submodel, now that we have the spatially heterogeneous interfacial current densities

Extended composite model for lead-acid. Uses leading-order model from pybamm.lead\_acid.LOQS

#### **Parameters**

- options (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).

Extends: pybamm.lead acid.BaseHigherOrderModel

#### **Full Model**

**class** pybamm.lead\_acid.**Full** (*options=None*, *name='Full model'*, *build=True*)

Porous electrode model for lead-acid, from<sup>1</sup>, based on the Newman-Tiedemann model.

#### **Parameters**

- **options** (dict, optional) A dictionary of options to be passed to the model.
- name (str, optional) The name of the model.
- **build** (bool, optional) Whether to build the model on instantiation. Default is True. Setting this option to False allows users to change any number of the submodels before building the complete model (submodels cannot be changed after the model is built).

<sup>&</sup>lt;sup>1</sup> V Sulzer, SJ Chapman, CP Please, DA Howey, and CW Monroe. Faster lead-acid battery simulations from porous-electrode theory: Part II. Asymptotic analysis. Journal of The Electrochemical Society 166.12 (2019), A2372–A2382.

#### References

Extends: pybamm.lead\_acid.BaseModel

class pybamm.lead\_acid.BasicFull(name='Basic full model')

Porous electrode model for lead-acid, from<sup>2</sup>.

This class differs from the pybamm.lead\_acid.Full model class in that it shows the whole model in a single class. This comes at the cost of flexibility in comparing different physical effects, and in general the main DFN class should be used instead.

**Parameters name** (str, optional) – The name of the model.

#### References

Extends: pybamm.lead\_acid.BaseModel

new copy (build=False)

Create a copy of the model. Overwrites the functionality of pybamm.BaseModel to make sure that the submodels are updated correctly

### 3.2.4 Submodels

#### **Base Submodel**

class pybamm.BaseSubModel (param, domain=None, name='Unnamed submodel', external=False)

The base class for all submodels. All submodels inherit from this class and must only provide public methods which overwrite those in this base class. Any methods added to a submodel that do not overwrite those in this bass class are made private with the prefix '\_', providing a consistent public interface for all submodels.

**Parameters** param (parameter class) - The model parameter symbols

### param

The model parameter symbols

Type parameter class

rhs

A dictionary that maps expressions (variables) to expressions that represent the rhs

Type dict

# algebraic

A dictionary that maps expressions (variables) to expressions that represent the algebraic equations. The algebraic expressions are assumed to equate to zero. Note that all the variables in the model must exist in the keys of *rhs* or *algebraic*.

Type dict

### initial conditions

A dictionary that maps expressions (variables) to expressions that represent the initial conditions for the state variables y. The initial conditions for algebraic variables are provided as initial guesses to a root finding algorithm that calculates consistent initial conditions.

Type dict

<sup>&</sup>lt;sup>2</sup> V Sulzer, SJ Chapman, CP Please, DA Howey, and CW Monroe. Faster lead-acid battery simulations from porous-electrode theory: Part II. Asymptotic analysis. Journal of The Electrochemical Society 166.12 (2019), A2372–A2382...

#### boundary conditions

A dictionary that maps expressions (variables) to expressions that represent the boundary conditions

Type dict

#### variables

A dictionary that maps strings to expressions that represent the useful variables

Type dict

#### events

A list of events. Each event can either cause the solver to terminate (e.g. concentration goes negative), or be used to inform the solver of the existance of a discontinuity (e.g. discontinuity in the input current)

Type list

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

#### get\_external\_variables()

A public method that returns the variables in a submodel which are supplied by an external source.

**Returns** A list of the external variables in the model.

Return type list

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

#### set algebraic(variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set\_events (variables)

A method to set events related to the state of submodel variable. Note: this method modifies the state of self.events. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### **Current Collector**

#### **Base Model**

```
class pybamm.current_collector.BaseModel(param)
```

Base class for current collector submodels

Parameters param (parameter class) – The parameters to use for this submodel

**Extends:** pybamm.BaseSubModel

### **Composite Potential Pair models**

```
class pybamm.current_collector.BaseCompositePotentialPair(param)
```

Composite potential pair model for the current collectors. This is identical to the BasePotentialPair model, except the name of the fundamental variables are changed to avoid clashes with leading order.

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.current\_collector.BasePotentialPair

#### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

#### Return type dict

```
class pybamm.current_collector.CompositePotentialPair2plus1D(param)
class pybamm.current collector.CompositePotentialPair1plus1D(param)
```

### **Effective Current collector Resistance models**

A model which calculates the effective Ohmic resistance of the current collectors in the limit of large electrical

conductivity. For details see<sup>1</sup>. Note that this formulation assumes uniform *potential* across the tabs. See pybamm.AlternativeEffectiveResistance2D for the formulation that assumes a uniform *current* density at the tabs (in 1D the two formulations are equivalent).

#### **Parameters**

- options (dict) A dictionary of options to be passed to the model. The options that can be set are listed below.
  - "dimensionality" [int, optional] Sets the dimension of the current collector problem.
     Can be 1 (default) or 2.
- name (str, optional) The name of the model.

#### References

Extends: pybamm.BaseModel

#### default solver

Return default solver based on whether model is ODE model or DAE model

```
post_process (solution, param_values, V_av, I_av)
```

Calculates the potentials in the current collector and the terminal voltage given the average voltage and current. Note: This takes in the *processed* V\_av and I\_av from a 1D simulation representing the average cell behaviour and returns a dictionary of processed potentials.

```
class pybamm.current collector.AlternativeEffectiveResistance2D
```

A model which calculates the effective Ohmic resistance of the 2D current collectors in the limit of large electrical conductivity. This model assumes a uniform *current density* at the tabs and the solution is computed by first solving and auxilliary problem which is the related to the resistances.

Extends: pybamm.BaseModel

#### default solver

Return default solver based on whether model is ODE model or DAE model

```
post_process (solution, param_values, V_av, I_av)
```

Calculates the potentials in the current collector given the average voltage and current. Note: This takes in the *processed* V\_av and I\_av from a 1D simulation representing the average cell behaviour and returns a dictionary of processed potentials.

### Uniform

```
class pybamm.current_collector.Uniform(param)
```

A submodel for uniform potential in the current collectors which is valid in the limit of fast conductivity in the current collectors.

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.current\_collector.BaseModel

```
get_coupled_variables (variables)
```

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

<sup>&</sup>lt;sup>1</sup> R Timms, SG Marquis, V Sulzer, CP Please and SJ Chapman. "Asymptotic Reduction of a Lithium-ion Pouch Cell Model". Submitted, 2020.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

#### **Potential Pair models**

### class pybamm.current\_collector.BasePotentialPair(param)

A submodel for Ohm's law plus conservation of current in the current collectors. For details on the potential pair formulation see<sup>1</sup> and<sup>2</sup>.

Parameters param (parameter class) - The parameters to use for this submodel

#### References

Extends: pybamm.current\_collector.BaseModel

#### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

#### set algebraic(variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

class pybamm.current\_collector.PotentialPair2plus1D(param)

Base class for a 2+1D potential pair model

### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

 ${\tt class} \ {\tt pybamm.current\_collector.PotentialPair1plus1D} \ ({\it param})$ 

Base class for a 1+1D potential pair model.

<sup>&</sup>lt;sup>1</sup> R Timms, SG Marquis, V Sulzer, CP Please and SJ Chapman. "Asymptotic Reduction of a Lithium-ion Pouch Cell Model". Submitted, 2020.

<sup>&</sup>lt;sup>2</sup> SG Marquis, R Timms, V Sulzer, CP Please and SJ Chapman. "A Suite of Reduced-Order Models of a Single-Layer Lithium-ion Pouch Cell". In preparation, 2020.

#### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### **Quite Conductive Potential Pair models**

#### class pybamm.current\_collector.BaseQuiteConductivePotentialPair(param)

A submodel for Ohm's law plus conservation of current in the current collectors, in the limit of quite conductive electrodes.

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.current\_collector.BaseModel

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

#### set\_algebraic (variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

```
class pybamm.current_collector.QuiteConductivePotentialPair1plus1D(param)
class pybamm.current_collector.QuiteConductivePotentialPair2plus1D(param)
```

#### Convection

The convection submodels are split up into "through-cell", which is the x-direction problem in the electrode domains, and "transverse", which is the z-direction problem in the separator domain

#### **Base Convection**

```
class pybamm.convection.BaseModel(param)
```

Base class for convection submodels.

 $\textbf{Parameters param} \ (\textit{parameter class}) - \textbf{The parameters to use for this submodel}$ 

Extends: pybamm.BaseSubModel

### **Through-cell Convection**

#### **Base Model**

**class** pybamm.convection.through\_cell.**BaseThroughCellModel**(param) Base class for convection submodels in the through-cell direction.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- \*\*Extends (\*\* pybamm.convection.BaseModel) -

#### No Convection

class pybamm.convection.through\_cell.NoConvection(param)
 A submodel for case where there is no convection.

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.convection.through\_cell.BaseThroughCellModel

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

#### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### Leading-Order Through-cell Model

```
class pybamm.convection.through_cell.Explicit(param)
    A submodel for the leading-order approximation of pressure-driven convection
```

**Parameters** param (parameter class) – The parameters to use for this submodel

Extends: pybamm.convection.through\_cell.BaseThroughCellModel

#### get coupled variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### **Full Through-cell Model**

```
class pybamm.convection.through_cell.Full (param)
```

Submodel for the full model of pressure-driven convection

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.convection.through cell.BaseThroughCellModel

#### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get fundamental variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set\_algebraic(variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubMode1.

**Parameters variables** (dict) – The variables in the whole model.

### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of

self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### **Transverse Convection**

#### **Base Model**

 $\textbf{class} \texttt{ pybamm.convection.transverse.BaseTransverseModel} \ (\textit{param})$ 

Base class for convection submodels in transverse directions.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- \*\*Extends (\*\* pybamm.convection.BaseModel) -

#### **No Transverse Convection**

class pybamm.convection.transverse.NoConvection(param)

Submodel for no convection in transverse directions

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- \*\*Extends (\*\* pybamm.convection.through\_cell.
  BaseTransverseModel)-

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### **Uniform Transverse Model**

```
class pybamm.convection.transverse.Uniform(param)
```

Submodel for uniform convection in transverse directions

**Parameters** param (parameter class) – The parameters to use for this submodel

Extends: pybamm.convection.through\_cell.BaseTransverseModel

```
get_coupled_variables (variables)
```

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### **Full Transverse Convection**

```
class pybamm.convection.transverse.Full(param)
```

Submodel for the full model of pressure-driven convection in transverse directions

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- \*\*Extends (\*\* pybamm.convection.through\_cell.
  BaseTransverseModel)-

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

#### set\_algebraic (variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set boundary conditions(variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **Electrode**

#### **Electrode Base Model**

**class** pybamm.electrode.**BaseElectrode**(param, domain, set\_positive\_potential=True)
Base class for electrode submodels.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) Either 'Negative' or 'Positive'
- **set\_positive\_potential** (bool, optional) If True the battery model sets the positive potential based on the current. If False, the potential is specified by the user. Default is True.
- \*\*Extends (\*\* pybamm.BaseSubModel) -

#### **Ohmic**

#### **Base Model**

class pybamm.electrode.ohm.BaseModel (param, domain, set\_positive\_potential=True)
A base class for electrode submodels that employ Ohm's law.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) Either 'Negative' or 'Positive'

Extends: pybamm.electrode.BaseElectrode

#### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **Leading Order Model**

**class** pybamm.electrode.ohm.**LeadingOrder** (*param*, *domain*, *set\_positive\_potential=True*)

An electrode submodel that employs Ohm's law the leading-order approximation to governing equations.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) Either 'Negative' or 'Positive'
- **set\_positive\_potential** (bool, optional) If True the battery model sets the positive potential based on the current. If False, the potential is specified by the user. Default is True.
- \*\*Extends(\*\* pybamm.electrode.ohm.BaseModel)-

### get\_coupled\_variables (variables)

Returns variables which are derived from the fundamental variables in the model.

#### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## **Composite Model**

#### class pybamm.electrode.ohm.Composite(param, domain)

An explicit composite leading and first order solution to solid phase current conservation with ohm's law. Note that the returned current density is only the leading order approximation.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) Either 'Negative electrode' or 'Positive electrode'
- \*\*Extends (\*\* pybamm.BaseOhm) -

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get fundamental variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **Full Model**

class pybamm.electrode.ohm.Full(param, domain)

Full model of electrode employing Ohm's law.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) Either 'Negative' or 'Positive'

Extends: pybamm.electrode.ohm.BaseModel

#### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set\_algebraic (variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

#### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **Surface Form**

#### class pybamm.electrode.ohm.SurfaceForm(param, domain)

A submodel for the electrode with Ohm's law in the surface potential formulation.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) Either 'Negative' or 'Positive'

Extends: pybamm.electrode.ohm.BaseModel

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### **Electrolyte Conductivity**

### **Base Electrolyte Conductivity Submodel**

class pybamm.electrolyte\_conductivity.BaseElectrolyteConductivity(param, domain=None)

Base class for conservation of charge in the electrolyte.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str, optional) The domain in which the model holds
- reactions (dict, optional) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.BaseSubModel) -

### set\_boundary\_conditions(variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **Leading Order Model**

class pybamm.electrolyte\_conductivity.LeadingOrder(param, domain=None)

Leading-order model for conservation of charge in the electrolyte employing the Stefan-Maxwell constitutive equations. (Leading refers to leading-order in the asymptotic reduction)

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str, optional) The domain in which the model holds
- reactions (dict, optional) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.electrolyte\_conductivity. BaseElectrolyteConductivity)-

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### **Composite Model**

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- higher\_order\_terms (str) What kind of higher-order terms to use ('composite' or 'first-order')
- domain (str, optional) The domain in which the model holds
- \*\*Extends (\*\* pybamm.electrolyte\_conductivity.

  BaseElectrolyteConductivity)-

#### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

#### **Full Model**

### class pybamm.electrolyte\_conductivity.Full(param)

Full model for conservation of charge in the electrolyte employing the Stefan-Maxwell constitutive equations. (Full refers to unreduced by asymptotic methods)

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.electrolyte\_conductivity. BaseElectrolyteConductivity)-

#### check\_algebraic\_equations (post\_discretisation)

Check that the algebraic equations are well-posed. Before discretisation, each algebraic equation key must appear in the equation After discretisation, there must be at least one StateVector in each algebraic equation

# ${\tt check\_default\_variables\_dictionaries}\ (\,)$

Chec that the right variables are provided.

#### check ics bcs()

Check that the initial and boundary conditions are well-posed.

#### check\_no\_repeated\_keys()

Check that no equation keys are repeated

# check\_well\_determined(post\_discretisation)

Check that the model is not under- or over-determined.

#### check\_well\_posedness(post\_discretisation=False)

Check that the model is well-posed by executing the following tests: - Model is not over- or underdetermined, by comparing keys and equations in rhs and algebraic. Overdetermined if more equations than variables, underdetermined if more variables than equations. - There is an initial condition in self.initial\_conditions for each variable/equation pair in self.rhs - There are appropriate boundary conditions in self.boundary\_conditions for each variable/equation pair in self.rhs and self.algebraic

**Parameters** post\_discretisation (boolean) - A flag indicating tests to be skipped after discretisation

#### default solver

Return default solver based on whether model is ODE model or DAE model

#### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

#### get\_external\_variables()

A public method that returns the variables in a submodel which are supplied by an external source.

**Returns** A list of the external variables in the model.

Return type list

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

### Return type dict

### info(symbol\_name)

Provides helpful summary information for a symbol.

Parameters parameter\_name(str)-

#### input\_parameters

Returns all the input parameters in the model

### new\_copy (build=False)

Create an empty copy with identical options, or new options if specified. The 'build' parameter is included for compatibility with subclasses, but unused.

#### parameters

Returns all the parameters in the model

### set\_algebraic(variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubMode1.

**Parameters variables** (dict) – The variables in the whole model.

#### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set events(variables)

A method to set events related to the state of submodel variable. Note: this method modifies the state of self.events. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### timescale

Timescale of model, to be used for non-dimensionalising time when solving

### update (\*submodels)

Update model to add new physics from submodels

Parameters submodel (iterable of pybamm. BaseModel) - The submodels from which to create new model

#### **Surface Form**

### **Full Model**

class pybamm.electrolyte\_conductivity.surface\_potential\_form.FullDifferential(param, do-

main)

Full model for conservation of charge in the electrolyte employing the Stefan-Maxwell constitutive equations and where capacitance is present. (Full refers to unreduced by asymptotic methods)

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.electrolyte\_conductivity.surface\_potential\_form.BaseFull

### set rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

class pybamm.electrolyte\_conductivity.surface\_potential\_form.FullAlgebraic(param, do-

Full model for conservation of charge in the electrolyte employing the Stefan-Maxwell constitutive equations. (Full refers to unreduced by asymptotic methods)

Parameters param – The parameters to use for this submodel

#### set algebraic(variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## **Leading Order Model**

class pybamm.electrolyte\_conductivity.surface\_potential\_form.LeadingOrderDifferential(param

domain)

Leading-order model for conservation of charge in the electrolyte employing the Stefan-Maxwell constitutive equations employing the surface potential difference formulation and where capacitance is present.

Parameters param (parameter class) - The parameters to use for this submodel

Extends: BaseLeadingOrderSurfaceForm

set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

main)

Leading-order model for conservation of charge in the electrolyte employing the Stefan-Maxwell constitutive equations employing the surface potential difference formulation.

Parameters param (parameter class) - The parameters to use for this submodel

Extends: BaseLeadingOrderSurfaceForm

```
set_algebraic(variables)
```

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

### **Electrolyte Diffusion**

### **Base Electrolyte Diffusion Submodel**

**class** pybamm.electrolyte\_diffusion.**BaseElectrolyteDiffusion**(param) Base class for conservation of mass in the electrolyte.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict, optional) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.BaseSubModel) -

#### set events(variables)

A method to set events related to the state of submodel variable. Note: this method modifies the state of self.events. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in <code>pybamm.BaseSubModel</code>.

**Parameters variables** (dict) – The variables in the whole model.

#### **Constant Concentration**

 ${\tt class} \ {\tt pybamm.electrolyte\_diffusion.ConstantConcentration} \ ({\it param})$ 

Class for constant concentration of electrolyte

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.electrolyte\_diffusion.BaseElectrolyteDiffusion

#### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### **Leading Order Model**

### class pybamm.electrolyte\_diffusion.LeadingOrder(param)

Class for conservation of mass in the electrolyte employing the Stefan-Maxwell constitutive equations. (Leading refers to leading order of asymptotic reduction)

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.electrolyte\_diffusion. BaseElectrolyteDiffusion)-

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

# Return type dict

### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

# **Composite Model**

# class pybamm.electrolyte\_diffusion.Composite(param, extended=False)

Class for conservation of mass in the electrolyte employing the Stefan-Maxwell constitutive equations. (Composite refers to composite model by asymptotic methods)

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- extended (bool) Whether to include feedback from the first-order terms
- \*\*Extends (\*\* pybamm.electrolyte\_diffusion.

  BaseElectrolyteDiffusion)-

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# set\_boundary\_conditions(variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of

self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set rhs(variables)

Composite reaction-diffusion with source terms from leading order

## **Full Model**

# class pybamm.electrolyte\_diffusion.Full(param)

Class for conservation of mass in the electrolyte employing the Stefan-Maxwell constitutive equations. (Full refers to unreduced by asymptotic methods)

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.electrolyte\_diffusion.

  BaseElectrolyteDiffusion)-

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

# get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

```
set rhs(variables)
```

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **External circuit**

Models to enforce different boundary conditions (as imposed by an imaginary external circuit) such as constant current, constant voltage, constant power, or any other relationship between the current and voltage. "Current control" enforces these directly through boundary conditions, while "Function control" submodels add an algebraic equation (for the current) and hence can be used to set any variable to be constant.

#### **Current control external circuit**

```
class pybamm.external_circuit.CurrentControl(param)
```

External circuit with current control.

# get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# **Function control external circuit**

**class** pybamm.external\_circuit.**FunctionControl**(*param*, *external\_circuit\_function*) External circuit with an arbitrary function.

```
get_fundamental_variables()
```

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# set\_algebraic (variables)

A method to set the differential equations which do not contain a time derivative. Note: this method

modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

```
set initial conditions(variables)
```

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

class pybamm.external\_circuit.VoltageFunctionControl(param)

External circuit with voltage control, implemented as an extra algebraic equation.

class pybamm.external\_circuit.PowerFunctionControl(param)
 External circuit with power control.

## **Experiment events**

class pybamm.external\_circuit.ExperimentEvents(param)

Model to impose the events for experiments.

```
set events(variables)
```

A method to set events related to the state of submodel variable. Note: this method modifies the state of self.events. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## Interface

# **Interface Base Model**

**class** pybamm.interface.**BaseInterface**(param, domain, reaction)

Base class for interfacial currents

Parameters

- param (parameter class) The parameters to use for this submodel
- **domain** (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- reaction (str) The name of the reaction being implemented
- \*\*Extends (\*\* pybamm.BaseSubModel) -

### **Kinetics**

class pybamm.interface.BaseKinetics(param, domain, reaction, options=None)
 Base submodel for kinetics

# **Parameters**

- param model parameters
- domain (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- reaction (str) The name of the reaction being implemented

- **options** (dict) A dictionary of options to be passed to the model. In this case "sei film resistance" is the important option. See <code>pybamm.BaseBatteryModel</code>
- \*\*Extends (\*\* pybamm.interface.BaseInterface) -

## get coupled variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

## set\_algebraic(variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set initial conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

**class** pybamm.interface.**ButlerVolmer** (param, domain, reaction, options=None)

Base submodel which implements the forward Butler-Volmer equation:

$$j = 2 * j_0(c) * \sinh((ne/(2 * (1 + \Theta T)) * \eta_r(c))$$

## **Parameters**

- param model parameters
- domain (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- reaction (str) The name of the reaction being implemented
- **options** (dict) A dictionary of options to be passed to the model. In this case "sei film resistance" is the important option. See <code>pybamm.BaseBatteryModel</code>
- \*\*Extends (\*\* pybamm.interface.kinetics.BaseKinetics) -

class pybamm.interface.NoReaction(param, domain, reaction)

Base submodel for when no reaction occurs

# Parameters

- param model parameters
- **domain** (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- reaction (str) The name of the reaction being implemented
- \*\*Extends (\*\* pybamm.interface.kinetics.BaseKinetics) -

**class** pybamm.interface.**ForwardTafel** (param, domain, reaction, options=None)

Base submodel which implements the forward Tafel equation:

$$j = j_0(c) * \exp((ne/(2 * (1 + \Theta T)) * \eta_r(c)))$$

### **Parameters**

- param model parameters
- domain (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- reaction (str) The name of the reaction being implemented
- **options** (dict) A dictionary of options to be passed to the model. In this case "sei film resistance" is the important option. See <code>pybamm.BaseBatteryModel</code>
- \*\*Extends (\*\* pybamm.interface.kinetics.BaseKinetics) -

**class** pybamm.interface.**BackwardTafel** (*param*, *domain*, *reaction*)

Base submodel which implements the backward Tafel equation:

$$j = -j_0(c) * \exp(-\eta_r(c))$$

#### **Parameters**

- param model parameters
- domain (str) The domain to implement the model, either: 'Negative' or 'Positive'.

Extends: pybamm.interface.kinetics.BaseKinetics

# **Inverse Kinetics**

A submodel that implements the inverted form of the Butler-Volmer relation to solve for the reaction overpotential.

# **Parameters**

- param Model parameters
- **domain** (*iter* of *str*, *optional*) The domain(s) in which to compute the interfacial current. Default is None, in which case j.domain is used.
- reaction (str) The name of the reaction being implemented
- **options** (dict) A dictionary of options to be passed to the model. In this case "sei film resistance" is the important option. See pybamm.BaseBatteryModel
- \*\*Extends (\*\* pybamm.interface.BaseInterface) -

### get coupled variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### **First-order Kinetics**

class pybamm.interface.FirstOrderKinetics (param, domain, leading\_order\_model)
 First-order kinetics

#### **Parameters**

- param model parameters
- domain (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- **leading\_order\_model** (pybamm.interface.kinetics.BaseKinetics) The leading-order model with respect to which this is first-order
- \*\*Extends (\*\* pybamm.interface.BaseInterface) -

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

Base inverse first-order kinetics. This class needs to consider *all* of the leading-order submodels simultaneously in order to find the first-order correction to the potentials

### **Parameters**

- param model parameters
- domain (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- leading\_order\_models (pybamm.interface.kinetics.BaseKinetics) The leading-order models with respect to which this is first-order
- \*\*Extends (\*\* pybamm.interface.BaseInterface) -

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## **Diffusion-limited**

class pybamm.interface.DiffusionLimited (param, domain, reaction, order)
Submodel for diffusion-limited kinetics

#### **Parameters**

- param model parameters
- **domain** (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- reaction (str) The name of the reaction being implemented
- **order** (*str*) The order of the model ("leading" or "full")
- \*\*Extends (\*\* pybamm.interface.BaseInterface) -

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## **SEI** models

class pybamm.sei.BaseModel (param, domain)
Base class for SEI models.

## Parameters

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain to implement the model, either: 'Negative' or 'Positive'.
- \*\*Extends (\*\* pybamm.interface.BaseInterface) -

class pybamm.sei.ConstantSEI(param, domain)

Base class for SEI with constant thickness.

Note that there is no SEI current, so we don't need to update the "sum of interfacial current densities" variables from pybamm.interface.BaseInterface

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'
- \*\*Extends (\*\* pybamm.sei.BaseModel) -

### get coupled variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# class pybamm.sei.ElectronMigrationLimited(param, domain)

Base class for electron-migration limited SEI growth.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- **domain** (str) The domain of the model either 'Negative' or 'Positive'
- \*\*Extends (\*\* pybamm.sei.BaseModel) -

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get fundamental variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### **set rhs** (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## class pybamm.sei.InterstitialDiffusionLimited(param, domain)

Base class for interstitial-diffusion limited SEI growth.

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'
- \*\*Extends (\*\* pybamm.sei.BaseModel) -

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get fundamental variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

class pybamm.sei.NoSEI(param, domain)

Base class for no SEI.

## **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'
- \*\*Extends (\*\* pybamm.sei.BaseModel) -

### get coupled variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get fundamental variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# class pybamm.sei.ReactionLimited(param, domain)

Base class for reaction limited SEI growth.

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'
- \*\*Extends (\*\* pybamm.sei.BaseModel) -

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get fundamental variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### **set rhs** (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### class pybamm.sei.SolventDiffusionLimited(param, domain)

Base class for solvent-diffusion limited SEI growth.

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'
- \*\*Extends (\*\* pybamm.sei.BaseModel) -

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get fundamental variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

## Return type dict

### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### class pybamm.sei.EcReactionLimited(param, domain)

Base class for reaction limited SEI growth.

# **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'
- \*\*Extends (\*\* pybamm.sei.BaseModel) -

### get coupled variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# set\_algebraic(variables)

A method to set the differential equations which do not contain a time derivative. Note: this method modifies the state of self.algebraic. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

# **Oxygen Diffusion**

## **Base Model**

class pybamm.oxygen\_diffusion.BaseModel(param)

Base class for conservation of mass of oxygen.

## **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict, optional) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.BaseSubModel) -

# **Composite Model**

## **class** pybamm.oxygen\_diffusion.**Composite** (param, extended=False)

Class for conservation of mass of oxygen. (Composite refers to composite expansion in asymptotic methods) In this model, extremely fast oxygen kinetics in the negative electrode imposes zero oxygen concentration there, and so the oxygen variable only lives in the separator and positive electrode. The boundary condition at the negative electrode/ separator interface is homogeneous Dirichlet.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- **extended** (bool) Whether to include feedback from the first-order terms
- \*\*Extends (\*\* pybamm.oxygen\_diffusion.Full) -

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

```
set rhs(variables)
```

Composite reaction-diffusion with source terms from leading order

### **First-Order Model**

```
class pybamm.oxygen_diffusion.FirstOrder(param)
```

Class for conservation of mass of oxygen. (First-order refers to first-order expansion in asymptotic methods) In this model, extremely fast oxygen kinetics in the negative electrode imposes zero oxygen concentration there, and so the oxygen variable only lives in the separator and positive electrode. The boundary condition at the negative electrode/ separator interface is homogeneous Dirichlet.

# **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.oxygen\_diffusion.BaseModel) -

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## **Full Model**

## class pybamm.oxygen\_diffusion.Full(param)

Class for conservation of mass of oxygen. (Full refers to unreduced by asymptotic methods) In this model, extremely fast oxygen kinetics in the negative electrode imposes zero oxygen concentration there, and so the oxygen variable only lives in the separator and positive electrode. The boundary condition at the negative electrode/ separator interface is homogeneous Dirichlet.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.oxygen\_diffusion.BaseModel) -

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

## set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set initial conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

# set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

# **Leading Order Model**

## class pybamm.oxygen\_diffusion.LeadingOrder(param)

Class for conservation of mass of oxygen. (Leading refers to leading order of asymptotic reduction)

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- reactions (dict) Dictionary of reaction terms
- \*\*Extends (\*\* pybamm.oxgen\_diffusion.BaseModel) -

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# set\_initial\_conditions (variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

# No Oxygen

## class pybamm.oxygen\_diffusion.NoOxygen(param)

Class for when there is no oxygen

Parameters param (parameter class) – The parameters to use for this submodel

Extends: pybamm.oxygen\_diffusion.BaseModel

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of

whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### **Particle**

# **Particle Base Model**

class pybamm.particle.BaseParticle(param, domain)

Base class for molar conservation in particles.

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'

Extends: pybamm.BaseSubModel

set\_events (variables)

A method to set events related to the state of submodel variable. Note: this method modifies the state of self.events. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## **Fickian Single Particle**

## class pybamm.particle.FickianSingleParticle(param, domain)

Base class for molar conservation in a single x-averaged particle which employs Fick's law.

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'

Extends: pybamm.particle.BaseParticle

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels

# Return type dict

# set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set initial conditions (variables)

For single particle models, initial conditions can't depend on x so we arbitrarily set the initial values of the single particles to be given by the values at x=0 in the negative electrode and x=1 in the positive electrode. Typically, supplied initial conditions are uniform x.

### set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## **Fickian Many Particles**

## class pybamm.particle.FickianManyParticles(param, domain)

Base class for molar conservation in many particles which employs Fick's law.

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'

Extends: pybamm.particle.BaseParticle

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

# get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set initial conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

# **Fast Single Particle**

# class pybamm.particle.FastSingleParticle(param, domain)

Base class for molar conservation in a single x-averaged particle with uniform concentration in r (i.e. infinitely fast diffusion within particles).

#### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'

Extends: pybamm.particle.BaseParticle

# get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

#### set initial conditions (variables)

For single particle models, initial conditions can't depend on x so we arbitrarily evaluate them at x=0 in the negative electrode and x=1 in the positive electrode (they will usually be constant)

# set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

# **Fast Many Particles**

## class pybamm.particle.FastManyParticles(param, domain)

Base class for molar conservation in many particles with uniform concentration in r (i.e. infinitely fast diffusion within particles).

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- domain (str) The domain of the model either 'Negative' or 'Positive'

Extends: pybamm.particle.BaseParticle

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

## set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters** variables (dict) – The variables in the whole model.

# **Porosity**

### **Base Model**

```
class pybamm.porosity.BaseModel(param)
```

Base class for porosity

Parameters param(parameter class) - The parameters to use for this submodel

Extends: pybamm.BaseSubModel

# set events (variables)

A method to set events related to the state of submodel variable. Note: this method modifies the state of self.events. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

# **Constant Porosity**

```
class pybamm.porosity.Constant(param)
```

Submodel for constant porosity

**Parameters param** (parameter class) – The parameters to use for this submodel

Extends: pybamm.porosity.BaseModel

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# **Leading-Order Model**

## class pybamm.porosity.LeadingOrder(param)

Leading-order model for reaction-driven porosity changes

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.porosity.BaseModel

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this

method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### **Full Model**

**class** pybamm.porosity.**Full**(param)

Full model for reaction-driven porosity changes

Parameters param (parameter class) – The parameters to use for this submodel

Extends: pybamm.porosity.BaseModel

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get fundamental variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubMode1.

**Parameters variables** (dict) – The variables in the whole model.

## **Thermal**

# **Base Thermal**

class pybamm.thermal.BaseThermal(param, cc\_dimension=0)

Base class for thermal effects

**Parameters** 

- param (parameter class) The parameters to use for this submodel
- cc\_dimension (int, optional) The dimension of the current collectors. Can be 0 (default), 1 or 2.
- \*\*Extends (\*\* pybamm.BaseSubModel) -

### **Isothermal Model**

class pybamm.thermal.isothermal.Isothermal(param)
 Class for isothermal submodel.

Parameters param (parameter class) - The parameters to use for this submodel

Extends: pybamm.thermal.BaseThermal

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

### get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

# **Lumped Model**

```
class pybamm.thermal.lumped.Lumped(param, cc_dimension=0, geometry='arbitrary')
    Class for lumped thermal submodel
```

### **Parameters**

- param (parameter class) The parameters to use for this submodel
- cc\_dimension (int, optional) The dimension of the current collectors. Can be 0 (default), 1 or 2.
- **geometry** (*string*, *optional*) The geometry for the lumped thermal submodel. Can be "arbitrary" (default) or pouch.
- \*\*Extends (\*\* pybamm.thermal.BaseThermal) -

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get fundamental variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

### set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

#### One Dimensional Model

# class pybamm.thermal.x\_full.OneDimensionalX(param)

Class for one-dimensional (x-direction) thermal submodel. Note: this model assumes infinitely large electrical and thermal conductivity in the current collectors, so that the contribution to the Ohmic heating from the current collectors is zero and the boundary conditions are applied at the edges of the electrodes (at x=0 and x=1, in non-dimensional coordinates).

## **Parameters**

- param (parameter class) The parameters to use for this submodel
- \*\*Extends (\*\* pybamm.thermal.BaseThermal) -

#### get coupled variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters** variables (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of

whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

## set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set\_rhs (variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **Pouch Cell**

# Thermal Model for "1+1D" Pouch Cell

# class pybamm.thermal.pouch\_cell.CurrentCollector1D(param)

Class for one-dimensional thermal submodel for use in the "1+1D" pouch cell model. The thermal model is averaged in the x-direction and is therefore referred to as 'x-lumped'. For more information see<sup>1</sup> and<sup>2</sup>.

Parameters param (parameter class) - The parameters to use for this submodel

### References

Extends: pybamm.thermal.BaseThermal

### get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

<sup>&</sup>lt;sup>1</sup> R Timms, SG Marquis, V Sulzer, CP Please and SJ Chapman. "Asymptotic Reduction of a Lithium-ion Pouch Cell Model". In preparation, 2020

<sup>&</sup>lt;sup>2</sup> SG Marquis, R Timms, V Sulzer, CP Please and SJ Chapman. "A Suite of Reduced-Order Models of a Single-Layer Lithium-ion Pouch Cell". In preparation, 2020.

### get fundamental variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

## set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## Thermal Model for "2+1D" Pouch Cell

# class pybamm.thermal.pouch\_cell.CurrentCollector2D(param)

Class for two-dimensional thermal submodel for use in the "2+1D" pouch cell model. The thermal model is averaged in the x-direction and is therefore referred to as 'x-lumped'. For more information see and and a second s

Parameters param (parameter class) - The parameters to use for this submodel

## References

Extends: pybamm.thermal.BaseThermal

# get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

**Return type** dict

<sup>&</sup>lt;sup>1</sup> R Timms, SG Marquis, V Sulzer, CP Please and SJ Chapman. "Asymptotic Reduction of a Lithium-ion Pouch Cell Model". In preparation, 2020.

<sup>&</sup>lt;sup>2</sup> SG Marquis, R Timms, V Sulzer, CP Please and SJ Chapman. "A Suite of Reduced-Order Models of a Single-Layer Lithium-ion Pouch Cell". In preparation, 2020.

## get\_fundamental\_variables()

A public method that creates and returns the variables in a submodel which can be created independent of other submodels. For example, the electrolyte concentration variables can be created independent of whether any other variables have been defined in the model. As a rule, if a variable can be created without variables from other submodels, then it should be placed in this method.

**Returns** The variables created by the submodel which are independent of variables in other submodels.

Return type dict

## set\_boundary\_conditions (variables)

A method to set the boundary conditions for the submodel. Note: this method modifies the state of self.boundary\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

## set\_initial\_conditions(variables)

A method to set the initial conditions for the submodel. Note: this method modifies the state of self.initial\_conditions. Unless overwritten by a submodel, the default behaviour of 'pass' is used a implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### set rhs(variables)

A method to set the right hand side of the differential equations which contain a time derivative. Note: this method modifies the state of self.rhs. Unless overwritten by a submodel, the default behaviour of 'pass' is used as implemented in pybamm.BaseSubModel.

**Parameters variables** (dict) – The variables in the whole model.

### **Tortuosity**

# **Base Model**

```
class pybamm.tortuosity.BaseModel(param, phase)
    Base class for tortuosity
```

# Parameters

- param (parameter class) The parameters to use for this submodel
- **phase** (str) The material for the model ('electrolyte' or 'electrode').
- \*\*Extends (\*\* pybamm.BaseSubModel) -

# **Bruggeman Model**

```
class pybamm.tortuosity.Bruggeman (param, phase, set_leading_order=False)
    Submodel for Bruggeman tortuosity
```

Extends: pybamm.tortuosity.BaseModel

## get\_coupled\_variables (variables)

A public method that creates and returns the variables in a submodel which require variables in other submodels to be set first. For example, the exchange current density requires the concentration in the electrolyte to be created before it can be created. If a variable can be created independent of other submodels then it should be created in 'get\_fundamental\_variables' instead of this method.

**Parameters variables** (dict) – The variables in the whole model.

**Returns** The variables created in this submodel which depend on variables in other submodels.

Return type dict

# 3.3 Parameters

# 3.3.1 Base Parameter Values

```
class pybamm.ParameterValues(values=None, chemistry=None)
```

The parameter values for a simulation.

Note that this class does not inherit directly from the python dictionary class as this causes issues with saving and loading simulations.

#### **Parameters**

- **values** (dict or string) Explicit set of parameters, or reference to a file of parameters If string, gets passed to read\_parameters\_csv to read a file.
- **chemistry** (dict) Dict of strings for default chemistries. Must be of the form: {"base chemistry": base\_chemistry, "cell": cell\_properties\_authorYear, "anode": anode\_chemistry\_authorYear, "separator": separator\_chemistry\_authorYear, "cathode": cathode\_chemistry\_authorYear, "electrolyte": electrolyte\_chemistry\_authorYear, "experiment": experimental\_conditions\_authorYear}. Then the anode chemistry is loaded from the file inputs/parameters/base\_chemistry/anodes/anode\_chemistry\_authorYear, etc. Parameters in "cell" should include geometry and current collector properties. Parameters in "experiment" should include parameters relating to experimental conditions, such as initial conditions and currents.

# **Examples**

```
>>> import pybamm
>>> values = {"some parameter": 1, "another parameter": 2}
>>> param = pybamm.ParameterValues(values)
>>> param["some parameter"]
1
>>> file = "input/parameters/lithium-ion/cells/kokam_Marquis2019/parameters.csv"
>>> values_path = pybamm.get_parameters_filepath(file)
>>> param = pybamm.ParameterValues(values=values_path)
>>> param["Negative current collector thickness [m]"]
2.5e-05
>>> param = pybamm.ParameterValues(chemistry=pybamm.parameter_sets.Marquis2019)
>>> param["Reference temperature [K]"]
298.15
```

#### copy()

Returns a copy of the parameter values. Makes sure to copy the internal dictionary.

# evaluate(symbol)

Process and evaluate a symbol.

Parameters symbol (pybamm. Symbol) - Symbol or Expression tree to evaluate

**Returns** The evaluated symbol

## **Return type** number of array

# static find\_parameter(path)

Look for parameter file in the different locations in PARAMETER\_PATH

### get (key, default=None)

Return item correspoonding to key if it exists, otherwise return default

#### items()

Get the items of the dictionary

### keys()

Get the keys of the dictionary

## print\_evaluated\_parameters (evaluated\_parameters, output\_file)

Print a dictionary of evaluated parameters to an output file

#### **Parameters**

- evaluated\_parameters (defaultdict) The evaluated parameters, for further processing if needed
- **output\_file** (*string*, *optional*) The file to print parameters to. If None, the parameters are not printed, and this function simply acts as a test that all the parameters can be evaluated

## print\_parameters (parameters, output\_file=None)

Return dictionary of evaluated parameters, and optionally print these evaluated parameters to an output file. For dimensionless parameters that depend on the C-rate, the value is given as a function of the C-rate (either x \* Crate or x / Crate depending on the dependence)

#### **Parameters**

- parameters (class or dict containing pybamm. Parameter objects) Class or dictionary containing all the parameters to be evaluated
- **output\_file** (*string*, *optional*) The file to print parameters to. If None, the parameters are not printed, and this function simply acts as a test that all the parameters can be evaluated, and returns the dictionary of evaluated parameters.

**Returns evaluated\_parameters** – The evaluated parameters, for further processing if needed **Return type** defaultdict

# **Notes**

A C-rate of 1 C is the current required to fully discharge the battery in 1 hour, 2 C is current to discharge the battery in 0.5 hours, etc

# $process\_boundary\_conditions(model)$

Process boundary conditions for a model Boundary conditions are dictionaries {"left": left bc, "right": right bc} in general, but may be imposed on the tabs (or *not* on the tab) for a small number of variables, e.g. {"negative tab": neg. tab bc, "positive tab": pos. tab bc "no tab": no tab bc}.

### process\_geometry (geometry)

Assign parameter values to a geometry (inplace).

Parameters geometry (dict) - Geometry specs to assign parameter values to

# process\_model (unprocessed\_model, inplace=True)

Assign parameter values to a model. Currently inplace, could be changed to return a new model.

### **Parameters**

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- unprocessed\_model (pybamm.BaseModel) Model to assign parameter values for
- inplace (bool, optional) If True, replace the parameters in the model in place. Otherwise, return a new model with parameter values set. Default is True.

Raises pybamm. ModelError - If an empty model is passed (model.rhs = {} and model.algebraic = {} and model.variables = {})

## process symbol(symbol)

Walk through the symbol and replace any Parameter with a Value. If a symbol has already been processed, the stored value is returned.

Parameters symbol (pybamm. Symbol) - Symbol or Expression tree to set parameters for

**Returns** symbol – Symbol with Parameter instances replaced by Value

Return type pybamm. Symbol

## read\_parameters\_csv (filename)

Reads parameters from csv file into dict.

**Parameters filename** (str) – The name of the csv file containing the parameters.

**Returns** {name: value} pairs for the parameters.

Return type dict

## search (key, print\_values=True)

Search dictionary for keys containing 'key'.

See pybamm.FuzzyDict.search().

**update** (*values*, *check\_conflict=False*, *check\_already\_exists=True*, *path=*")

Update parameter dictionary, while also performing some basic checks.

## **Parameters**

- values (dict) Dictionary of parameter values to update parameter dictionary with
- **check\_conflict** (bool, optional) Whether to check that a parameter in *values* has not already been defined in the parameter class when updating it, and if so that its value does not change. This is set to True during initialisation, when parameters are combined from different sources, and is False by default otherwise
- **check\_already\_exists** (bool, optional) Whether to check that a parameter in *values* already exists when trying to update it. This is to avoid cases where an intended change in the parameters is ignored due a typo in the parameter name, and is True by default but can be manually overridden.
- path (string, optional) Path from which to load functions

# update\_from\_chemistry(chemistry)

Load standard set of components from a 'chemistry' dictionary

values()

Get the values of the dictionary

# 3.3.2 Geometric Parameters

Standard geometric parameters

# 3.3.3 Electrical Parameters

# 3.3.4 Thermal Parameters

## 3.3.5 Standard Lithium-ion Parameters

Standard parameters for lithium-ion battery models

## 3.3.6 Standard Lead-Acid Parameters

Standard Parameters for lead-acid battery models

# 3.3.7 Parameters Sets

Parameter sets from papers. The 'citation' entry provides a reference to the appropriate paper in the file "py-bamm/CITATIONS.txt". To see which parameter sets have been used in your simulation, add the line "py-bamm.print\_citations()" to your script.

# Lithium-ion parameter sets

- Chen2020: C.-H. Chen, F. Brosa Planella, K. O'Regan, D. Gastol, W. D. Widanage, and E. Kendrick. "Development of Experimental Techniques for Parameterization of Multi-scale Lithium-ion Battery Models."
   Journal of the Electrochemical Society, 167(8), 080534 (2020).
- Ecker2015: M. Ecker, T. K. D. Tran, P. Dechent, S. Käbitz, A. Warnecke, and D. U. Sauer. "Parameterization of a Physico-Chemical Model of a Lithium-Ion Battery. I. Determination of Parameters." Journal of the Electrochemical Society, 162(9), A1836-A1848 (2015).
- Marquis2019: S. G. Marquis, V. Sulzer, R. Timms, C. P. Please and S. J. Chapman. "An asymptotic derivation of a single particle model with electrolyte." Journal of the Electrochemical Society, 166(15), A3693–A3706 (2019).
- Mohtat2020: Submitted for publication.
- NCA\_Kim2011: G. H. Kim, K. Smith, K. J. Lee, S. Santhanagopalan, and A. Pesaran. "Multi-domain modeling of lithium-ion batteries encompassing multi-physics in varied length scales." Journal of The Electrochemical Society, 158(8), A955-A969 (2011).
- Ramadass 2004: P. Ramadass, B. Haran, P. M. Gomadam, R. White, and B. N. Popov. "Development of First Principles Capacity Fade Model for Li-Ion Cells." Journal of the Electrochemical Society, 151(2), A196-A203 (2004).

# Lead-acid parameter sets

• Sulzer2019: V. Sulzer, S. J. Chapman, C. P. Please, D. A. Howey, and C. W. Monroe, "Faster lead-acid battery simulations from porous-electrode theory: Part I. Physical model." Journal of the Electrochemical Society, 166(12), 2363 (2019).

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# 3.4 Geometry

# 3.4.1 Geometry

```
class pybamm.Geometry (geometry)
```

A geometry class to store the details features of the cell geometry.

The values assigned to each domain are dictionaries containing the spatial variables in that domain, along with expression trees giving their min and maximum extents. For example, the following dictionary structure would represent a Geometry with a single domain "negative electrode", defined using the variable  $x_n$  which has a range from 0 to the pre-defined parameter  $l_n$ .

```
{"negative electrode": {x_n: {"min": pybamm.Scalar(0), "max": l_n}}}
```

Extends: dict

Parameters geometries (dict) - The dictionary to create the geometry with

parameters

Returns all the parameters in the geometry

# 3.4.2 Battery Geometry

pybamm.battery\_geometry (include\_particles=True, current\_collector\_dimension=0)
A convenience function to create battery geometries.

## **Parameters**

- include\_particles (bool) Whether to include particle domains
- current\_collector\_dimensions (int, default) The dimensions of the current collector. Should be 0 (default), 1 or 2

**Returns** A geometry class for the battery

Return type pybamm. Geometry

# 3.5 Meshes

### **3.5.1 Meshes**

class pybamm.Mesh(geometry, submesh\_types, var\_pts)

Mesh contains a list of submeshes on each subdomain.

Extends: dict

### **Parameters**

- **geometry** contains the geometry of the problem.
- **submesh\_types** (dict) contains the types of submeshes to use (e.g. Uniform1DSubMesh)
- **submesh\_pts** (dict) contains the number of points on each subdomain

### add\_ghost\_meshes()

Create meshes for potential ghost nodes on either side of each submesh, using self.submeshclass This will be useful for calculating the gradient with Dirichlet BCs.

#### combine submeshes (\*submeshnames)

Combine submeshes into a new submesh, using self.submeshclass Raises pybamm.DomainError if submeshes to be combined do not match up (edges are not aligned).

Parameters submeshnames (list of str) - The names of the submeshes to be combined

**Returns** submesh – A new submesh with the class defined by self.submeshclass

Return type self.submeshclass

## class pybamm.SubMesh

Base submesh class. Contains the position of the nodes, the number of mesh points, and (optionally) information about the tab locations.

class pybamm.MeshGenerator(submesh\_type, submesh\_params=None)

Base class for mesh generator objects that are used to generate submeshes.

### **Parameters**

- **submesh\_type** (*pybamm.SubMesh*) The type of submesh to use (e.g. Uniform1DSubMesh).
- **submesh\_params** (dict, optional) Contains any parameters required by the submesh

## 3.5.2 0D Sub Mesh

class pybamm.SubMesh0D (position, npts=None)

0D submesh class. Contains the position of the node.

#### **Parameters**

- **position** (dict) A dictionary that contains the position of the 0D submesh (a signle point) in space
- npts (dict, optional) Number of points to be used. Included for compatibility with other meshes, but ignored by this mesh class
- \*\*Extends ("": pybamm.SubMesh) -

### 3.5.3 1D Sub Meshes

class pybamm.SubMesh1D(edges, coord\_sys, tabs=None)

1D submesh class. Contains the position of the nodes, the number of mesh points, and (optionally) information about the tab locations.

# **Parameters**

- **edges** (array\_like) An array containing the points corresponding to the edges of the submesh
- **coord\_sys** (string) The coordinate system of the submesh
- tabs (dict, optional) A dictionary that contains information about the size and location of the tabs
- \*\*Extends ("": pybamm.SubMesh) -

class pybamm.Uniform1DSubMesh (lims, npts)

A class to generate a uniform submesh on a 1D domain

### **Parameters**

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- lims (dict) A dictionary that contains the limits of the spatial variables
- npts (dict) A dictionary that contains the number of points to be used on each spatial variable. Note: the number of nodes (located at the cell centres) is npts, and the number of edges is npts+1.
- \*\*Extends ("": pybamm.SubMesh1D) -

class pybamm. Exponential1DSubMesh (lims, npts, side='symmetric', stretch=None)

A class to generate a submesh on a 1D domain in which the points are clustered close to one or both of boundaries using an exponential formula on the interval [a,b].

If side is "left", the gridpoints are given by

+ a, for k = 1, ..., N, where N is the number of nodes.

Is side is "right", the gridpoints are given by

+ a, for k = 1, ..., N.

If side is "symmetric", the first half of the interval is meshed using the gridpoints

+ a, for k = 1, ..., N. The grid spacing is then reflected to contruct the grid on the full interval [a,b].

In the above, alpha is a stretching factor. As the number of gridpoints tends to infinity, the ratio of the largest and smallest grid cells tends to exp(alpha).

#### **Parameters**

- lims (dict) A dictionary that contains the limits of the spatial variables
- npts (dict) A dictionary that contains the number of points to be used on each spatial variable. Note: the number of nodes (located at the cell centres) is npts, and the number of edges is npts+1.
- **side** (*str*, *optional*) Whether the points are clustered near to the left or right boundary, or both boundaries. Can be "left", "right" or "symmetric". Default is "symmetric"
- **stretch** (*float*, *optional*) The factor (alpha) which appears in the exponential. If side is "symmetric" then the default stretch is 1.15. If side is "left" or "right" then the default stretch is 2.3.
- \*\*Extends ("": pybamm.SubMesh1D) -

**class** pybamm.**Chebyshev1DSubMesh** (*lims*, *npts*, *tabs=None*)

A class to generate a submesh on a 1D domain using Chebyshev nodes on the interval (a, b), given by

$$x_k = \frac{1}{2}(a+b) + \frac{1}{2}(b-a)\cos(\frac{2k-1}{2N}\pi),$$

for k = 1, ..., N, where N is the number of nodes. Note: this mesh then appends the boundary edges, so that the mesh edges are given by

$$a < x_1 < ... < x_N < b$$
.

### **Parameters**

• lims (dict) - A dictionary that contains the limits of the spatial variables

- npts (dict) A dictionary that contains the number of points to be used on each spatial variable. Note: the number of nodes (located at the cell centres) is npts, and the number of edges is npts+1.
- tabs (dict, optional) A dictionary that contains information about the size and location of the tabs
- \*\*Extends ("": pybamm.SubMesh1D) -

## class pybamm.UserSupplied1DSubMesh(lims, npts, edges=None)

A class to generate a submesh on a 1D domain from a user supplied array of edges.

### **Parameters**

- lims (dict) A dictionary that contains the limits of the spatial variables
- npts (dict) A dictionary that contains the number of points to be used on each spatial variable. Note: the number of nodes (located at the cell centres) is npts, and the number of edges is npts+1.
- **edges** (array\_like) The array of points which correspond to the edges of the mesh.
- \*\*Extends ("": pybamm.SubMesh1D) -

# 3.5.4 2D Sub Meshes

# class pybamm.ScikitSubMesh2D (edges, coord\_sys, tabs)

2D submesh class. Contains information about the 2D finite element mesh. Note: This class only allows for the use of piecewise-linear triangular finite elements.

### **Parameters**

- edges (array\_like) An array containing the points corresponding to the edges of the submesh
- **coord\_sys** (*string*) The coordinate system of the submesh
- tabs (dict, optional) A dictionary that contains information about the size and location of the tabs
- \*\*Extends ("": pybamm.SubMesh) -

### on\_boundary (y, z, tab)

A method to get the degrees of freedom corresponding to the subdomains for the tabs.

# class pybamm.ScikitUniform2DSubMesh (lims, npts)

Contains information about the 2D finite element mesh with uniform grid spacing (can be different spacing in y and z). Note: This class only allows for the use of piecewise-linear triangular finite elements.

### **Parameters**

- lims (dict) A dictionary that contains the limits of each spatial variable
- npts (dict) A dictionary that contains the number of points to be used on each spatial variable
- \*\*Extends ("": pybamm.ScikitSubMesh2D) -

# class pybamm.ScikitExponential2DSubMesh (lims, npts, side='top', stretch=2.3)

Contains information about the 2D finite element mesh generated by taking the tensor product of a uniformly spaced grid in the y direction, and a unequally spaced grid in the z direction in which the points are clustered

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close to the top boundary using an exponential formula on the interval [a,b]. The gridpoints in the z direction are given by

+ a, for k = 1, ..., N, where N is the number of nodes. Here alpha is a stretching factor. As the number of gridpoints tends to infinity, the ratio of the largest and smallest grid cells tends to exp(alpha).

Note: in the future this will be extended to allow points to be clustered near any of the boundaries.

## **Parameters**

- lims (dict) A dictionary that contains the limits of each spatial variable
- npts (dict) A dictionary that contains the number of points to be used on each spatial variable
- **side** (*str*, *optional*) Whether the points are clustered near to a particular boundary. At present, can only be "top". Default is "top".
- **stretch** (*float*, *optional*) The factor (alpha) which appears in the exponential. Default is 2.3.
- \*\*Extends ("": pybamm.ScikitSubMesh2D) -

### class pybamm.ScikitChebyshev2DSubMesh (lims, npts)

Contains information about the 2D finite element mesh generated by taking the tensor product of two 1D meshes which use Chebyshev nodes on the interval (a, b), given by

$$x_k = \frac{1}{2}(a+b) + \frac{1}{2}(b-a)\cos(\frac{2k-1}{2N}\pi),$$

for k = 1, ..., N, where N is the number of nodes. Note: this mesh then appends the boundary edgess, so that the 1D mesh edges are given by

$$a < x_1 < ... < x_N < b$$
.

Note: This class only allows for the use of piecewise-linear triangular finite elements.

#### **Parameters**

- lims (dict) A dictionary that contains the limits of each spatial variable
- npts (dict) A dictionary that contains the number of points to be used on each spatial variable
- \*\*Extends ("": pybamm.ScikitSubMesh2D) -

## class pybamm. UserSupplied2DSubMesh (lims, npts, y\_edges=None, z\_edges=None)

A class to generate a tensor product submesh on a 2D domain by using two user supplied vectors of edges: one for the y-direction and one for the z-direction. Note: this mesh should be created using UserSupplied2DSubMeshGenerator.

# **Parameters**

- lims (dict) A dictionary that contains the limits of the spatial variables
- npts (dict) A dictionary that contains the number of points to be used on each spatial variable. Note: the number of nodes (located at the cell centres) is npts, and the number of edges is npts+1.
- **y\_edges** (array\_like) The array of points which correspond to the edges in the y direction of the mesh.
- z\_edges (array\_like) The array of points which correspond to the edges in the z direction of the mesh.
- \*\*Extends ("": pybamm.ScikitSubMesh2D) -

# 3.6 Discretisation and spatial methods

### 3.6.1 Discretisation

### class pybamm.Discretisation(mesh=None, spatial\_methods=None)

The discretisation class, with methods to process a model and replace Spatial Operators with Matrices and Variables with StateVectors

#### **Parameters**

- mesh (pybamm. Mesh) contains all submeshes to be used on each domain
- **spatial\_methods** (dict) a dictionary of the spatial methods to be used on each domain. The keys correspond to the model domains and the values to the spatial method.

### check\_initial\_conditions (model)

Check initial conditions are a numpy array

### check\_initial\_conditions\_rhs (model)

Check initial conditions and rhs have the same shape

#### check model (model)

Perform some basic checks to make sure the discretised model makes sense.

### check tab conditions(symbol, bcs)

Check any boundary conditions applied on "negative tab", "positive tab" and "no tab". For 1D current collector meshes, these conditions are converted into boundary conditions on "left" (tab at z=0) or "right" (tab at z=1) depending on the tab location stored in the mesh. For 2D current collector meshes, the boundary conditions can be applied on the tabs directly.

### **Parameters**

- **symbol** (pybamm.expression\_tree.symbol.Symbol) The symbol on which the boundary conditions are applied.
- **bcs** (dict) The dictionary of boundary conditions (a dict of {side: equation}).

**Returns** The dictionary of boundary conditions, with the keys changed to "left" and "right" where necessary.

### Return type dict

#### check variables(model)

Check variables in variable list against rhs Be lenient with size check if the variable in model.variables is broadcasted, or a concatenation (if broadcasted, variable is a multiplication with a vector of ones)

#### create\_jacobian (model)

Creates Jacobian of the discretised model. Note that the model is assumed to be of the form  $M*y\_dot = f(t,y)$ , where M is the (possibly singular) mass matrix. The Jacobian is df/dy.

Note: At present, calculation of the Jacobian is deferred until after simplification, since it is much faster to compute the Jacobian of the simplified model. However, in some use cases (e.g. running the same model multiple times but with different parameters) it may be more efficient to compute the Jacobian once, before simplification, so that parameters in the Jacobian can be updated (see PR #670).

**Parameters model** (pybamm.BaseModel) – Discretised model. Must have attributes rhs, initial\_conditions and boundary\_conditions (all dicts of {variable: equation})

**Returns** The expression trees corresponding to the Jacobian of the model

Return type pybamm. Concatenation

#### create mass matrix(model)

Creates mass matrix of the discretised model. Note that the model is assumed to be of the form  $M*y\_dot = f(t,y)$ , where M is the (possibly singular) mass matrix.

**Parameters model** (*pybamm.BaseModel*) – Discretised model. Must have attributes rhs, initial\_conditions and boundary\_conditions (all dicts of {variable: equation})

#### Returns

- pybamm. Matrix The mass matrix
- pybamm.Matrix The inverse of the ode part of the mass matrix (required by solvers which only accept the ODEs in explicit form)

### process\_boundary\_conditions (model)

Discretise model boundary\_conditions, also converting keys to ids

**Parameters model** (pybamm.BaseModel) – Model to dicretise. Must have attributes rhs, initial\_conditions and boundary\_conditions (all dicts of {variable: equation})

**Returns** Dictionary of processed boundary conditions

Return type dict

### process\_dict (var\_eqn\_dict)

Discretise a dictionary of {variable: equation}, broadcasting if necessary (can be model.rhs, model.algebraic, model.initial\_conditions or model.variables).

**Parameters var\_eqn\_dict** (dict) – Equations ({variable: equation} dict) to dicretise (can be model.rhs, model.algebraic, model.initial conditions or model.variables)

**Returns** new\_var\_eqn\_dict – Discretised equations

Return type dict

### process initial conditions (model)

Discretise model initial conditions.

**Parameters model** (pybamm.BaseModel) – Model to dicretise. Must have attributes rhs, initial\_conditions and boundary\_conditions (all dicts of {variable: equation})

**Returns** Tuple of processed\_initial\_conditions (dict of initial conditions) and concatenated\_initial\_conditions (numpy array of concatenated initial conditions)

Return type tuple

# $\verb|process_model| (model, inplace = True, check\_model = True)|$

Discretise a model. Currently inplace, could be changed to return a new model.

#### **Parameters**

- model (pybamm.BaseModel) Model to dicretise. Must have attributes rhs, initial\_conditions and boundary\_conditions (all dicts of {variable: equation})
- inplace (bool, optional) If True, discretise the model in place. Otherwise, return a new discretised model. Default is True.
- **check\_model** (bool, optional) If True, model checks are performed after discretisation. For large systems these checks can be slow, so can be skipped by setting this option to False. When developing, testing or debugging it is recommend to leave this option as True as it may help to identify any errors. Default is True.

**Returns model\_disc** – The discretised model. Note that if inplace is True, model will have also been discretised in place so model == model\_disc. If inplace is False, model != model disc

#### Return type pybamm.BaseModel

**Raises** pybamm.ModelError – If an empty model is passed ( $model.rhs = \{\}$ ) and  $model.algebraic = \{\}$  and  $model.variables = \{\}$ )

### process\_rhs\_and\_algebraic (model)

Discretise model equations - differential ('rhs') and algebraic.

**Parameters model** (pybamm.BaseModel) – Model to dicretise. Must have attributes rhs, initial\_conditions and boundary\_conditions (all dicts of {variable: equation})

**Returns** Tuple of processed\_rhs (dict of processed differential equations), processed\_concatenated\_rhs, processed\_algebraic (dict of processed algebraic equations) and processed\_concatenated\_algebraic

### Return type tuple

# $process\_symbol(symbol)$

Discretise operators in model equations. If a symbol has already been discretised, the stored value is returned.

Parameters symbol (pybamm.expression\_tree.symbol.Symbol) - Symbol to discretise

**Returns** Discretised symbol

Return type pybamm.expression\_tree.symbol.Symbol

### set external variables (model)

Add external variables to the list of variables to account for, being careful about concatenations

### set\_internal\_boundary\_conditions (model)

A method to set the internal boundary conditions for the submodel. These are required to properly calculate the gradient. Note: this method modifies the state of self.boundary\_conditions.

### set\_variable\_slices(variables)

Sets the slicing for variables.

Parameters variables (iterable of pybamm. Variables) - The variables for which to set slices

# 3.6.2 Spatial Method

#### class pybamm.SpatialMethod(options=None)

A general spatial methods class, with default (trivial) behaviour for some spatial operations. All spatial methods will follow the general form of SpatialMethod in that they contain a method for broadcasting variables onto a mesh, a gradient operator, and a divergence operator.

Parameters mesh – Contains all the submeshes for discretisation

### boundary\_integral (child, discretised\_child, region)

Implements the boundary integral for a spatial method.

- child (pybamm. Symbol) The symbol to which is being integrated
- discretised\_child (pybamm.Symbol) The discretised symbol of the correct size
- **region** (str) The region of the boundary over which to integrate. If region is None (default) the integration is carried out over the entire boundary. If region is *negative tab*

or *positive tab* then the integration is only carried out over the appropriate part of the boundary corresponding to the tab.

**Returns** Contains the result of acting the discretised boundary integral on the child discretised\_symbol

Return type class: pybamm.Array

### boundary\_value\_or\_flux (symbol, discretised\_child, bcs=None)

Returns the boundary value or flux using the approriate expression for the spatial method. To do this, we create a sparse vector 'bv\_vector' that extracts either the first (for side="left") or last (for side="right") point from 'discretised\_child'.

#### **Parameters**

- **symbol** (*pybamm*. *Symbol*) The boundary value or flux symbol
- discretised\_child (pybamm.StateVector) The discretised variable from which to calculate the boundary value
- **bcs** (dict (optional)) The boundary conditions. If these are supplied and "use bcs" is True in the options, then these will be used to improve the accuracy of the extrapolation.

**Returns** The variable representing the surface value.

Return type pybamm. Matrix Multiplication

broadcast (symbol, domain, auxiliary\_domains, broadcast\_type)

Broadcast symbol to a specified domain.

#### **Parameters**

- **symbol** (pybamm.Symbol) The symbol to be broadcasted
- domain (iterable of strings) The domain to broadcast to
- auxiliary\_domains (dict of strings) The auxiliary domains for broadcasting
- **broadcast\_type** (*str*) The type of broadcast: 'primary to node', 'primary to edges', 'secondary to nodes', 'secondary to edges', 'full to nodes' or 'full to edges'

**Returns broadcasted\_symbol** – The discretised symbol of the correct size for the spatial method

Return type class: pybamm.Symbol

### concatenation(disc\_children)

Discrete concatenation object.

**Parameters disc\_children** (list) – List of discretised children

**Returns** Concatenation of the discretised children

Return type pybamm.DomainConcatenation

# ${\tt delta\_function}\ (symbol,\ discretised\_symbol)$

Implements the delta function on the approriate side for a spatial method.

- symbol (pybamm. Symbol) The symbol to which is being integrated
- discretised\_symbol (pybamm.Symbol) The discretised symbol of the correct size

**divergence** (symbol, discretised\_symbol, boundary\_conditions)
Implements the divergence for a spatial method.

#### **Parameters**

- **symbol** (pybamm.Symbol) The symbol that we will take the gradient of.
- discretised\_symbol (pybamm.Symbol) The discretised symbol of the correct size
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"left": left bc, "right": right bc}})

**Returns** Contains the result of acting the discretised divergence on the child discretised\_symbol

**Return type** class: *pybamm.Array* 

gradient (symbol, discretised\_symbol, boundary\_conditions)

Implements the gradient for a spatial method.

#### **Parameters**

- **symbol** (pybamm.Symbol) The symbol that we will take the gradient of.
- discretised\_symbol (pybamm.Symbol) The discretised symbol of the correct size
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"left": left bc, "right": right bc}})

**Returns** Contains the result of acting the discretised gradient on the child discretised\_symbol

Return type class: pybamm.Array

gradient\_squared(symbol, discretised\_symbol, boundary\_conditions)

Implements the inner product of the gradient with itself for a spatial method.

### **Parameters**

- **symbol** (pybamm. Symbol) The symbol that we will take the gradient of.
- discretised\_symbol (pybamm.Symbol) The discretised symbol of the correct size
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"left": left bc, "right": right bc}})

**Returns** Contains the result of taking the inner product of the result of acting the discretised gradient on the child discretised\_symbol with itself

**Return type** class: pybamm.Array

indefinite\_integral (child, discretised\_child, direction)

Implements the indefinite integral for a spatial method.

#### **Parameters**

- child (pybamm. Symbol) The symbol to which is being integrated
- discretised\_child (pybamm.Symbol) The discretised symbol of the correct size
- **direction** (str) The direction of integration

**Returns** Contains the result of acting the discretised indefinite integral on the child discretised\_symbol

#### **Return type** class: pybamm.Array

integral (child, discretised\_child, integration\_dimension)

Implements the integral for a spatial method.

#### **Parameters**

- child (pybamm. Symbol) The symbol to which is being integrated
- discretised\_child (pybamm.Symbol) The discretised symbol of the correct size
- integration\_dimension (str, optional) The dimension in which to integrate (default is "primary")

**Returns** Contains the result of acting the discretised integral on the child discretised\_symbol

Return type class: pybamm.Array

internal\_neumann\_condition (left\_symbol\_disc, right\_symbol\_disc, left\_mesh, right\_mesh)
 A method to find the internal neumann conditions between two symbols on adjacent subdomains.

#### **Parameters**

- left\_symbol\_disc (pybamm. Symbol) The discretised symbol on the left subdomain
- right\_symbol\_disc (pybamm. Symbol) The discretised symbol on the right subdomain
- **left\_mesh** (*list*) The mesh on the left subdomain
- right\_mesh (list) The mesh on the right subdomain

laplacian (symbol, discretised\_symbol, boundary\_conditions)

Implements the laplacian for a spatial method.

#### **Parameters**

- **symbol** (*pybamm*. *Symbol*) The symbol that we will take the gradient of.
- discretised\_symbol (pybamm.Symbol) The discretised symbol of the correct size
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"left": left bc, "right": right bc}})

**Returns** Contains the result of acting the discretised laplacian on the child discretised\_symbol

Return type class: pybamm.Array

mass\_matrix (symbol, boundary\_conditions)

Calculates the mass matrix for a spatial method.

#### **Parameters**

- **symbol** (*pybamm*. *Variable*) The variable corresponding to the equation for which we are calculating the mass matrix.
- **boundary\_conditions** (dict)—The boundary conditions of the model ({symbol.id: {"left": left bc, "right": right bc}})

**Returns** The (sparse) mass matrix for the spatial method.

Return type pybamm.Matrix

#### process\_binary\_operators (bin\_op, left, right, disc\_left, disc\_right)

Discretise binary operators in model equations. Default behaviour is to return a new binary operator with the discretised children.

#### **Parameters**

- bin\_op (pybamm.BinaryOperator) Binary operator to discretise
- left (pybamm. Symbol) The left child of bin\_op
- right (pybamm. Symbol) The right child of bin\_op
- disc\_left (pybamm.Symbol) The discretised left child of bin\_op
- disc\_right (pybamm.Symbol) The discretised right child of bin\_op

### **Returns** Discretised binary operator

Return type pybamm.BinaryOperator

### spatial\_variable(symbol)

Convert a pybamm. Spatial Variable node to a linear algebra object that can be evaluated (here, a pybamm. Vector on either the nodes or the edges).

**Parameters** symbol (pybamm. Spatial Variable) – The spatial variable to be discretised.

**Returns** Contains the discretised spatial variable

Return type pybamm. Vector

### 3.6.3 Finite Volume

### class pybamm.FiniteVolume(options=None)

A class which implements the steps specific to the finite volume method during discretisation.

For broadcast and mass\_matrix, we follow the default behaviour from SpatialMethod.

### **Parameters**

- mesh (pybamm. Mesh) Contains all the submeshes for discretisation
- \*\*Extends ("": pybamm.SpatialMethod) -

#### add ghost nodes (symbol, discretised symbol, bcs)

Add ghost nodes to a symbol.

For Dirichlet bcs, for a boundary condition "y = a at the left-hand boundary", we concatenate a ghost node to the start of the vector y with value "2\*a - y1" where y1 is the value of the first node. Similarly for the right-hand boundary condition.

For Neumann bcs no ghost nodes are added. Instead, the exact value provided by the boundary condition is used at the cell edge when calculating the gradient (see <code>pybamm.FiniteVolume.add\_neumann\_values())</code>.

- symbol (pybamm. Spatial Variable) The variable to be discretised
- discretised\_symbol (pybamm. Vector) Contains the discretised variable
- **bcs** (dict of tuples (*pybamm. Scalar*, str)) Dictionary (with keys "left" and "right") of boundary conditions. Each boundary condition consists of a value and a flag indicating its type (e.g. "Dirichlet")

**Returns** *Matrix* @ *discretised\_symbol* + *bcs\_vector*. When evaluated, this gives the discretised\_symbol, with appropriate ghost nodes concatenated at each end.

Return type pybamm. Symbol

### add\_neumann\_values (symbol, discretised\_gradient, bcs, domain)

Add the known values of the gradient from Neumann boundary conditions to the discretised gradient.

Dirichlet bcs are implemented using ghost nodes, see pybamm.FiniteVolume. add ghost nodes().

#### **Parameters**

- symbol (pybamm. Spatial Variable) The variable to be discretised
- discretised\_gradient (pybamm.Vector) Contains the discretised gradient of symbol
- **bcs** (dict of tuples (*pybamm.Scalar*, str)) Dictionary (with keys "left" and "right") of boundary conditions. Each boundary condition consists of a value and a flag indicating its type (e.g. "Dirichlet")
- **domain** (*list of strings*) The domain of the gradient of the symbol (may include ghost nodes)

**Returns** *Matrix* @ *discretised\_gradient* + *bcs\_vector*. When evaluated, this gives the discretised\_gradient, with the values of the Neumann boundary conditions concatenated at each end (if given).

Return type pybamm. Symbol

### boundary\_value\_or\_flux (symbol, discretised\_child, bcs=None)

Uses extrapolation to get the boundary value or flux of a variable in the Finite Volume Method.

See pybamm. Spatial Method. boundary\_value ()

### concatenation(disc\_children)

Discrete concatenation, taking <code>edge\_to\_node</code> for children that evaluate on edges. See <code>pybamm.SpatialMethod.concatenation()</code>

definite\_integral\_matrix(child, vector\_type='row', integration\_dimension='primary')

Matrix for finite-volume implementation of the definite integral in the primary dimension

$$I = \int_{a}^{b} f(s) \, ds$$

for where a and b are the left-hand and right-hand boundaries of the domain respectively

### **Parameters**

- child (pybamm. Symbol) The symbol being integrated
- **vector\_type** (*str*, *optional*) Whether to return a row or column vector in the primary dimension (default is row)
- integration\_dimension (str, optional) The dimension in which to integrate (default is "primary")

Returns The finite volume integral matrix for the domain

Return type pybamm. Matrix

### delta\_function (symbol, discretised\_symbol)

Delta function. Implemented as a vector whose only non-zero element is the first (if symbol.side = "left")

or last (if symbol.side = "right"), with appropriate value so that the integral of the delta function across the whole domain is the same as the integral of the discretised symbol across the whole domain.

See pybamm. Spatial Method. delta\_function()

### divergence (symbol, discretised\_symbol, boundary\_conditions)

Matrix-vector multiplication to implement the divergence operator. See pybamm.SpatialMethod.divergence()

#### divergence matrix(domains)

Divergence matrix for finite volumes in the appropriate domain. Equivalent to div(N) = (N[1:] - N[:-1])/dx

**Parameters domains** (dict) – The domain(s) and auxiliary domain in which to compute the divergence matrix

Returns The (sparse) finite volume divergence matrix for the domain

Return type pybamm. Matrix

## edge\_to\_node (discretised\_symbol, method='arithmetic')

Convert a discretised symbol evaluated on the cell edges to a discretised symbol evaluated on the cell nodes. See pybamm.FiniteVolume.shift()

#### gradient (symbol, discretised symbol, boundary conditions)

Matrix-vector multiplication to implement the gradient operator. See pybamm.SpatialMethod.gradient()

#### gradient\_matrix (domain, auxiliary\_domains)

Gradient matrix for finite volumes in the appropriate domain. Equivalent to grad(y) = (y[1:] - y[:-1])/dx

#### **Parameters**

- **domains** (list) The domain(s) in which to compute the gradient matrix, including ghost nodes
- auxiliary\_domains (dict) The auxiliary domains in which to compute the gradient matrix

Returns The (sparse) finite volume gradient matrix for the domain

Return type pybamm. Matrix

### indefinite\_integral (child, discretised\_child, direction)

Implementation of the indefinite integral operator.

## indefinite\_integral\_matrix\_edges (domains, direction)

Matrix for finite-volume implementation of the indefinite integral where the integrand is evaluated on mesh edges (shape (n+1, 1)). The integral will then be evaluated on mesh nodes (shape (n, 1)).

### **Parameters**

- **domains** (dict) The domain(s) and auxiliary domains of integration
- **direction** (str) The direction of integration (forward or backward). See notes.

**Returns** The finite volume integral matrix for the domain

Return type pybamm.Matrix

### **Notes**

### Forward integral

$$F(x) = \int_0^x f(u) \, du$$

The indefinite integral must satisfy the following conditions:

- F(0) = 0
- $f(x) = \frac{dF}{dx}$

or, in discrete form,

- BoundaryValue(F, "left") = 0, i.e.  $3 * F_0 F_1 = 0$
- $f_{i+1/2} = (F_{i+1} F_i)/dx_{i+1/2}$

Hence we must have

- $F_0 = du_{1/2} * f_{1/2}/2$
- $F_{i+1} = F_i + du_{i+1/2} * f_{i+1/2}$

Note that  $f_{-1/2}$  and  $f_{end+1/2}$  are included in the discrete integrand vector f, so we add a column of zeros at each end of the indefinite integral matrix to ignore these.

### **Backward integral**

$$F(x) = \int_{x}^{e} n df(u) du$$

The indefinite integral must satisfy the following conditions:

- F(end) = 0
- $f(x) = -\frac{dF}{dx}$

or, in discrete form,

- BoundaryValue(F, "right") = 0, i.e.  $3 * F_{end} F_{end-1} = 0$
- $f_{i+1/2} = -(F_{i+1} F_i)/dx_{i+1/2}$

Hence we must have

- $F_{end} = du_{end+1/2} * f_{end-1/2}/2$
- $F_{i-1} = F_i + du_{i-1/2} * f_{i-1/2}$

Note that  $f_{-1/2}$  and  $f_{end+1/2}$  are included in the discrete integrand vector f, so we add a column of zeros at each end of the indefinite integral matrix to ignore these.

#### indefinite integral matrix nodes (domains, direction)

Matrix for finite-volume implementation of the (backward) indefinite integral where the integrand is evaluated on mesh nodes (shape (n, 1)). The integral will then be evaluated on mesh edges (shape (n+1, 1)). This is just a straightforward (backward) cumulative sum of the integrand

#### **Parameters**

- domains (dict) The domain(s) and auxiliary domains of integration
- **direction** (str) The direction of integration (forward or backward)

Returns The finite volume integral matrix for the domain

Return type pybamm. Matrix

integral (child, discretised\_child, integration\_dimension)

Vector-vector dot product to implement the integral operator.

internal\_neumann\_condition(left\_symbol\_disc, right\_symbol\_disc, left\_mesh, right\_mesh)

A method to find the internal neumann conditions between two symbols on adjacent subdomains.

- left\_symbol\_disc (pybamm. Symbol) The discretised symbol on the left subdomain
- right\_symbol\_disc (pybamm. Symbol) The discretised symbol on the right subdomain
- left\_mesh (list) The mesh on the left subdomain
- right\_mesh (list) The mesh on the right subdomain

### laplacian (symbol, discretised\_symbol, boundary\_conditions)

Laplacian operator, implemented as div(grad(.)) See pybamm.SpatialMethod.laplacian()

### node\_to\_edge (discretised\_symbol, method='arithmetic')

Convert a discretised symbol evaluated on the cell nodes to a discretised symbol evaluated on the cell edges. See pybamm.FiniteVolume.shift()

### preprocess\_external\_variables(var)

For finite volumes, we need the boundary fluxes for discretising properly. Here, we extrapolate and then add them to the boundary conditions.

**Parameters var** (pybamm. Variable or pybamm. Concatenation) – The external variable that is to be processed

**Returns** new\_bcs – A dictionary containing the new boundary conditions

Return type dict

### process\_binary\_operators (bin\_op, left, right, disc\_left, disc\_right)

Discretise binary operators in model equations. Performs appropriate averaging of diffusivities if one of the children is a gradient operator, so that discretised sizes match up. For this averaging we use the harmonic mean [1].

[1] Recktenwald, Gerald. "The control-volume finite-difference approximation to the diffusion equation." (2012).

#### **Parameters**

- bin\_op (pybamm.BinaryOperator) Binary operator to discretise
- left (pybamm.Symbol) The left child of bin\_op
- right (pybamm. Symbol) The right child of bin\_op
- disc\_left (pybamm. Symbol) The discretised left child of bin\_op
- disc\_right (pybamm.Symbol) The discretised right child of bin\_op

**Returns** Discretised binary operator

Return type pybamm.BinaryOperator

### shift (discretised\_symbol, shift\_key, method)

Convert a discretised symbol evaluated at edges/nodes, to a discretised symbol evaluated at nodes/edges. Can be the arithmetic mean or the harmonic mean.

Note: when computing fluxes at cell edges it is better to take the harmonic mean based on [1].

[1] Recktenwald, Gerald. "The control-volume finite-difference approximation to the diffusion equation." (2012).

### **Parameters**

• discretised\_symbol (pybamm.Symbol) - Symbol to be averaged. When evaluated, this symbol returns either a scalar or an array of shape (n,) or (n+1,),

where n is the number of points in the mesh for the symbol's domain (n = self.mesh[symbol.domain].npts)

- **shift\_key** (*str*) Whether to shift from nodes to edges ("node to edge"), or from edges to nodes ("edge to node")
- **method** (str) Whether to use the "arithmetic" or "harmonic" mean

**Returns** Averaged symbol. When evaluated, this returns either a scalar or an array of shape (n+1,) (if *shift\_key* = "node to edge") or (n,) (if *shift\_key* = "edge to node")

Return type pybamm. Symbol

### spatial\_variable(symbol)

Creates a discretised spatial variable compatible with the FiniteVolume method.

Parameters symbol (pybamm. Spatial Variable) - The spatial variable to be discretised.

**Returns** Contains the discretised spatial variable

Return type pybamm. Vector

### 3.6.4 Scikit Finite Elements

### class pybamm.ScikitFiniteElement (options=None)

A class which implements the steps specific to the finite element method during discretisation. The class uses scikit-fem to discretise the problem to obtain the mass and stiffness matrices. At present, this class is only used for solving the Poisson problem -grad $^2$  u = f in the y-z plane (i.e. not the through-cell direction).

For broadcast we follow the default behaviour from SpatialMethod.

#### **Parameters**

- mesh (pybamm. Mesh) Contains all the submeshes for discretisation
- \*\*Extends ("": pybamm.SpatialMethod) -

assemble\_mass\_form (symbol, boundary\_conditions, region='interior')

Assembles the form of the finite element mass matrix over the domain interior or boundary.

#### **Parameters**

- **symbol** (*pybamm.Variable*) The variable corresponding to the equation for which we are calculating the mass matrix.
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"negative tab": neg. tab bc, "positive tab": pos. tab bc}})
- **region** (*str*, *optional*) The domain over which to assemble the mass matrix form. Can be "interior" (default) or "boundary".

**Returns** The (sparse) mass matrix for the spatial method.

Return type pybamm.Matrix

bc\_apply (M, boundary, zero=False)

Adjusts the assemled finite element matrices to account for boundary conditons.

- M (scipy.sparse.coo\_matrix) The assemled finite element matrix to adjust.
- boundary (numpy.array) Array of the indicies which correspond to the boundary.

• **zero** (bool, optional) – If True, the rows of M given by the indicies in boundary are set to zero. If False, the diagonal element is set to one. default is False.

### boundary\_integral (child, discretised\_child, region)

Implementation of the boundary integral operator. See pybamm.SpatialMethod.boundary\_integral()

### boundary\_integral\_vector(domain, region)

A node in the expression tree representing an integral operator over the boundary of a domain

$$I = \int_{\partial a} f(u) \, du,$$

where  $\partial a$  is the boundary of the domain, and  $u \in$  domain boundary.

#### **Parameters**

- domain (list) The domain(s) of the variable in the integrand
- **region** (str) The region of the boundary over which to integrate. If region is *entire* the integration is carried out over the entire boundary. If region is *negative tab* or *positive tab* then the integration is only carried out over the appropriate part of the boundary corresponding to the tab.

**Returns** The finite element integral vector for the domain

Return type pybamm. Matrix

#### boundary\_mass\_matrix (symbol, boundary\_conditions)

Calculates the mass matrix for the finite element method assembled over the boundary.

#### **Parameters**

- **symbol** (*pybamm.Variable*) The variable corresponding to the equation for which we are calculating the mass matrix.
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"negative tab": neg. tab bc, "positive tab": pos. tab bc}})

**Returns** The (sparse) mass matrix for the spatial method.

Return type pybamm. Matrix

### boundary\_value\_or\_flux (symbol, discretised\_child, bcs=None)

Returns the average value of the symbol over the negative tab ("negative tab") or the positive tab ("positive tab") in the Finite Element Method.

Overwrites the default pybamm. Spatial Method. boundary\_value()

### definite\_integral\_matrix(child, vector\_type='row')

Matrix for finite-element implementation of the definite integral over the entire domain

$$I = \int_{\Omega} f(s) \, dx$$

for where  $\Omega$  is the domain.

#### **Parameters**

- child (pybamm. Symbol) The symbol being integrated
- **vector\_type** (*str*, *optional*) Whether to return a row or column vector (default is row)

**Returns** The finite element integral vector for the domain

#### Return type pybamm. Matrix

### divergence (symbol, discretised\_symbol, boundary\_conditions)

Matrix-vector multiplication to implement the divergence operator. See pybamm.SpatialMethod.divergence()

#### gradient (symbol, discretised\_symbol, boundary\_conditions)

Matrix-vector multiplication to implement the gradient operator. The gradient w of the function u is approximated by the finite element method using the same function space as u, i.e. we solve w = grad(u), which corresponds to the weak form  $w^*v^*dx = \text{grad}(u)^*v^*dx$ , where v is a suitable test function.

#### **Parameters**

- **symbol** (*pybamm*. *Symbol*) The symbol that we will take the laplacian of.
- discretised\_symbol (pybamm.Symbol) The discretised symbol of the correct size
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"negative tab": neg. tab bc, "positive tab": pos. tab bc}})

**Returns** A concatenation that contains the result of acting the discretised gradient on the child discretised\_symbol. The first column corresponds to the y-component of the gradient and the second column corresponds to the z component of the gradient.

Return type class: pybamm.Concatenation

#### gradient\_matrix (symbol, boundary\_conditions)

Gradient matrix for finite elements in the appropriate domain.

#### **Parameters**

- symbol (pybamm.Symbol) The symbol for which we want to calculate the gradient matrix
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"negative tab": neg. tab bc, "positive tab": pos. tab bc}})

**Returns** The (sparse) finite element gradient matrix for the domain

Return type pybamm. Matrix

### gradient\_squared(symbol, discretised\_symbol, boundary\_conditions)

Multiplication to implement the inner product of the gradient operator with itself. See pybamm. SpatialMethod.gradient\_squared()

### indefinite\_integral (child, discretised\_child, direction)

Implementation of the indefinite integral operator. The input discretised child must be defined on the internal mesh edges. See pybamm.SpatialMethod.indefinite\_integral()

#### integral (child, discretised\_child, integration\_dimension)

Vector-vector dot product to implement the integral operator. See pybamm.SpatialMethod.
integral()

# ${\tt laplacian}\ (symbol,\ discretised\_symbol,\ boundary\_conditions)$

Matrix-vector multiplication to implement the laplacian operator.

- **symbol** (*pybamm*. *Symbol*) The symbol that we will take the laplacian of.
- discretised\_symbol (pybamm.Symbol) The discretised symbol of the correct size

• **boundary\_conditions** (dict) – The boundary conditions of the model ({symbol.id: {"negative tab": neg. tab bc, "positive tab": pos. tab bc}})

**Returns** Contains the result of acting the discretised gradient on the child discretised\_symbol

Return type class: pybamm.Array

#### mass\_matrix (symbol, boundary\_conditions)

Calculates the mass matrix for the finite element method.

#### **Parameters**

- **symbol** (*pybamm.Variable*) The variable corresponding to the equation for which we are calculating the mass matrix.
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"negative tab": neg. tab bc, "positive tab": pos. tab bc}})

**Returns** The (sparse) mass matrix for the spatial method.

Return type pybamm. Matrix

### spatial\_variable(symbol)

Creates a discretised spatial variable compatible with the FiniteElement method.

**Parameters** symbol (pybamm. Spatial Variable) – The spatial variable to be discretised.

**Returns** Contains the discretised spatial variable

Return type pybamm. Vector

#### stiffness matrix(symbol, boundary conditions)

Laplacian (stiffness) matrix for finite elements in the appropriate domain.

### **Parameters**

- **symbol** (*pybamm*. *Symbol*) The symbol for which we want to calculate the laplacian matrix
- **boundary\_conditions** (dict) The boundary conditions of the model ({symbol.id: {"negative tab": neg. tab bc, "positive tab": pos. tab bc}})

**Returns** The (sparse) finite element stiffness matrix for the domain

Return type pybamm. Matrix

# 3.6.5 Zero Dimensional Spatial Method

### class pybamm.ZeroDimensionalSpatialMethod(options=None)

A discretisation class for the zero dimensional mesh

#### **Parameters**

- mesh Contains all the submeshes for discretisation
- \*\*Extends\*\* (pybamm.SpatialMethod) -

## boundary\_value\_or\_flux (symbol, discretised\_child, bcs=None)

In OD, the boundary value is the identity operator. See SpatialMethod. boundary\_value\_or\_flux()

#### indefinite integral (child, discretised child, direction)

Calculates the zero-dimensional indefinite integral. If 'direction' is forward, this is the identity operator. If 'direction' is backward, this is the negation operator.

integral (child, discretised child, integration dimension)

Calculates the zero-dimensional integral, i.e. the identity operator

mass\_matrix (symbol, boundary\_conditions)

Calculates the mass matrix for a spatial method. Since the spatial method is zero dimensional, this is simply the number 1.

### 3.7 Solvers

### 3.7.1 Base Solver

class pybamm.BaseSolver(method=None, rtol=1e-06, atol=1e-06, root\_method=None, root\_tol=1e-06, max steps='deprecated')

Solve a discretised model.

#### **Parameters**

- method (str, optional) The method to use for integration, specific to each solver
- rtol (float, optional) The relative tolerance for the solver (default is 1e-6).
- atol (float, optional) The absolute tolerance for the solver (default is 1e-6).
- root\_method (str or pybamm algebraic solver class, optional) The method to use to find initial conditions (for DAE solvers). If a solver class, must be an algebraic solver class. If "casadi", the solver uses casadi's Newton rootfinding algorithm to find initial conditions. Otherwise, the solver uses 'scipy.optimize.root' with method specified by 'root\_method' (e.g. "lm", "hybr", ...)
- root\_tol (float, optional) The tolerance for the initial-condition solver (default is 1e-6).

### calculate\_consistent\_state (model, time=0, inputs=None)

Calculate consistent state for the algebraic equations through root-finding. model.y0 is used as the initial guess for rootfinding

#### **Parameters**

- model (pybamm. BaseModel) The model for which to calculate initial conditions.
- time (float) The time at which to calculate the states
- inputs (dict, optional) Any input parameters to pass to the model when solving

**Returns y0\_consistent** – Initial conditions that are consistent with the algebraic equations (roots of the algebraic equations)

**Return type** array-like, same shape as y0\_guess

copy()

Returns a copy of the solver

# ${\tt get\_termination\_reason}$ (solution, events)

Identify the cause for termination. In particular, if the solver terminated due to an event, (try to) pinpoint which event was responsible. Note that the current approach (evaluating all the events and then finding which one is smallest at the final timestep) is pretty crude, but is the easiest one that works for all the different solvers.

## **Parameters**

• solution (pybamm. Solution) - The solution object

• events (dict) – Dictionary of events

set up (model, inputs=None)

Unpack model, perform checks, simplify and calculate jacobian.

#### **Parameters**

- model (pybamm.BaseModel) The model whose solution to calculate. Must have attributes rhs and initial conditions
- inputs (dict, optional) Any input parameters to pass to the model when solving

**solve** (*model*, *t\_eval=None*, *external\_variables=None*, *inputs=None*)

Execute the solver setup and calculate the solution of the model at specified times.

### **Parameters**

- model (pybamm.BaseModel) The model whose solution to calculate. Must have attributes rhs and initial\_conditions
- t\_eval (numeric type) The times (in seconds) at which to compute the solution
- **external\_variables** (dict) A dictionary of external variables and their corresponding values at the current time
- inputs (dict, optional) Any input parameters to pass to the model when solving

**Raises** pybamm.ModelError – If an empty model is passed ( $model.rhs = \{\}$ ) and  $model.algebraic=\{\}$  and  $model.variables = \{\}$ )

 $\verb|step| (old\_solution, model|, dt, npts=2, external\_variables=None, inputs=None, save=True)|$ 

Step the solution of the model forward by a given time increment. The first time this method is called it executes the necessary setup by calling *self.set\_up(model)*.

#### **Parameters**

- old\_solution (pybamm. Solution or None) The previous solution to be added to. If *None*, a new solution is created.
- model (pybamm.BaseModel) The model whose solution to calculate. Must have attributes rhs and initial conditions
- dt (numeric type) The timestep (in seconds) over which to step the solution
- **npts** (*int*, *optional*) The number of points at which the solution will be returned during the step dt. default is 2 (returns the solution at t0 and t0 + dt).
- **external\_variables** (dict) A dictionary of external variables and their corresponding values at the current time
- inputs (dict, optional) Any input parameters to pass to the model when solving
- **save** (bool) Turn on to store the solution of all previous timesteps

**Raises** pybamm.ModelError - If an empty model is passed (model.rhs =  $\{\}$  and model.algebraic =  $\{\}$  and model.variables =  $\{\}$ )

# 3.7.2 Dummy Solver

### class pybamm.DummySolver

Dummy solver class for empty models.

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# 3.7.3 Scipy Solver

class pybamm. ScipySolver (method='BDF', rtol=1e-06, atol=1e-06, extra\_options=None) Solve a discretised model, using scipy.integrate.solve\_ivp.

#### **Parameters**

- method (str, optional) The method to use in solve\_ivp (default is "BDF")
- rtol (float, optional) The relative tolerance for the solver (default is 1e-6).
- **atol** (*float*, *optional*) The absolute tolerance for the solver (default is 1e-6).
- extra\_options (dict, optional) Any options to pass to the solver. Please consult SciPy documentation for details.

### 3.7.4 Scikits.odes Solvers

class pybamm. ScikitsOdeSolver (method='cvode', rtol=1e-06, atol=1e-06, linsolver='deprecated', extra\_options=None)

Solve a discretised model, using scikits.odes.

### **Parameters**

- **method** (str, optional) The method to use in solve\_ivp (default is "BDF")
- rtol (float, optional) The relative tolerance for the solver (default is 1e-6).
- atol (float, optional) The absolute tolerance for the solver (default is 1e-6).
- **extra\_options** (*dict*, *optional*) Any options to pass to the solver. Please consult scikits.odes documentation for details. Some common keys:
  - 'linsolver': can be 'dense' (= default), 'lapackdense', 'spgmr', 'spbcgs', 'sptfqmr'

class pybamm. ScikitsDaeSolver (method='ida', rtol=1e-06, atol=1e-06, root\_method='casadi', root\_tol=1e-06, extra\_options=None, max\_steps='deprecated') Solve a discretised model, using scikits.odes.

- method (str, optional) The method to use in solve\_ivp (default is "BDF")
- rtol (float, optional) The relative tolerance for the solver (default is 1e-6).
- atol (float, optional) The absolute tolerance for the solver (default is 1e-6).
- root\_method (str or pybamm algebraic solver class, optional) The method to use to find initial conditions (for DAE solvers). If a solver class, must be an algebraic solver class. If "casadi", the solver uses casadi's Newton rootfinding algorithm to find initial conditions. Otherwise, the solver uses 'scipy.optimize.root' with method specified by 'root\_method' (e.g. "lm", "hybr", ...)
- root\_tol (float, optional) The tolerance for the initial-condition solver (default is 1e-6).
- **extra\_options** (*dict*, *optional*) Any options to pass to the solver. Please consult scikits.odes documentation for details. Some common keys:
  - 'max\_steps': maximum (int) number of steps the solver can take

# 3.7.5 Casadi Solver

class pybamm. CasadiSolver (mode='safe', rtol=1e-06, atol=1e-06, root\_method='casadi', root\_tol=1e-06, max\_step\_decrease\_count=5, dt\_max=None, extra\_options\_setup=None, extra\_options\_call=None)

Solve a discretised model, using CasADi.

Extends: pybamm.BaseSolver

#### **Parameters**

- mode(str) How to solve the model (default is "safe"):
  - "fast": perform direct integration, without accounting for events. Recommended when simulating a drive cycle or other simulation where no events should be triggered.
  - "safe": perform step-and-check integration in global steps of size dt\_max, checking whether events have been triggered. Recommended for simulations of a full charge or discharge.
  - "old safe": perform step-and-check integration in steps of size dt for each dt in t\_eval, checking whether events have been triggered.
- **rtol** (*float*, *optional*) The relative tolerance for the solver (default is 1e-6).
- **atol** (*float*, *optional*) The absolute tolerance for the solver (default is 1e-6).
- root\_method (str or pybamm algebraic solver class, optional) The method to use to find initial conditions (for DAE solvers). If a solver class, must be an algebraic solver class. If "casadi", the solver uses casadi's Newton rootfinding algorithm to find initial conditions. Otherwise, the solver uses 'scipy.optimize.root' with method specified by 'root\_method' (e.g. "lm", "hybr", ...)
- root\_tol (float, optional) The tolerance for root-finding. Default is 1e-6.
- max\_step\_decrease\_counts (float, optional) The maximum number of times step size can be decreased before an error is raised. Default is 5.
- dt\_max (float, optional) The maximum global step size (in seconds) used in "safe" mode. If None the default value corresponds to a non-dimensional time of 0.01 (i.e. 0.01 \* model.timescale\_eval).
- extra\_options\_setup (dict, optional) Any options to pass to the CasADi integrator when creating the integrator. Please consult CasADi documentation for details. Some typical options:
  - "max\_num\_steps": Maximum number of integrator steps
- **extra\_options\_call** (*dict*, *optional*) Any options to pass to the CasADi integrator when calling the integrator. Please consult CasADi documentation for details.

# 3.7.6 Algebraic Solvers

class pybamm.AlgebraicSolver(method='lm', tol=1e-06, extra\_options=None)

Solve a discretised model which contains only (time independent) algebraic equations using a root finding algorithm. Uses scipy.optimize.root. Note: this solver could be extended for quasi-static models, or models in which the time derivative is manually discretised and results in a (possibly nonlinear) algebraic system at each time level.

#### **Parameters**

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- **method** (str, optional) The method to use to solve the system (default is "lm"). If it starts with "lsq", least-squares minimization is used. The method for least-squares can be specified in the form "lsq\_methodname"
- tol (float, optional) The tolerance for the solver (default is 1e-6).
- **extra\_options** (*dict*, *optional*) Any options to pass to the rootfinder. Vary depending on which method is chosen. Please consult SciPy documentation for details.

#### class pybamm. CasadiAlgebraicSolver (tol=1e-06, extra options=None)

Solve a discretised model which contains only (time independent) algebraic equations using CasADi's root finding algorithm. Note: this solver could be extended for quasi-static models, or models in which the time derivative is manually discretised and results in a (possibly nonlinear) algebraic system at each time level.

#### **Parameters**

- tol (float, optional) The tolerance for the solver (default is 1e-6).
- extra\_options (dict, optional) Any options to pass to the CasADi rootfinder.
   Please consult CasADi documentation for details.

# 3.7.7 Solutions

```
class pybamm._BaseSolution(t, y, t_event=None, y_event=None, termination='final time', copy this=None)
```

(Semi-private) class containing the solution of, and various attributes associated with, a PyBaMM model. This class is automatically created by the *Solution* class, and should never be called from outside the *Solution* class.

#### **Parameters**

- t (numpy.array, size (n,)) A one-dimensional array containing the times at which the solution is evaluated
- y (numpy.array, size (m, n)) A two-dimensional array containing the values of the solution. y[i, :] is the vector of solutions at time t[i].
- t\_event (numpy.array, size (1,)) A zero-dimensional array containing the time at which the event happens.
- **y\_event** (numpy.array, size (m,)) A one-dimensional array containing the value of the solution at the time when the event happens.
- **termination** (str) String to indicate why the solution terminated
- copy\_this (pybamm.Solution, optional) A solution to copy, if provided. Default is None.

### inputs

Values of the inputs

#### model

Model used for solution

#### save (filename)

Save the whole solution using pickle

 $\verb+save_data+ (filename, variables=None, to\_format='pickle')$ 

Save solution data only (raw arrays)

#### **Parameters**

• **filename** (str) – The name of the file to save data to

- variables (list, optional) List of variables to save. If None, saves all of the variables that have been created so far
- to\_format (str, optional) The format to save to. Options are:
  - 'pickle' (default): creates a pickle file with the data dictionary
  - 'matlab': creates a .mat file, for loading in matlab
  - 'csv': creates a csv file (1D variables only)

t

Times at which the solution is evaluated

#### t\_event

Time at which the event happens

#### termination

Reason for termination

### update (variables)

Add Processed Variables to the dictionary of variables in the solution

У

Values of the solution

#### y event

Value of the solution at the time of the event

**class** pybamm.**Solution**(*t*, *y*, *t\_event=None*, *y\_event=None*, *termination='final time'*)

Class extending the base solution, with additional functionality for concatenating different solutions together

```
Extends: _BaseSolution
```

append (solution, start\_index=1, create\_sub\_solutions=False)

Appends solution.t and solution.y onto self.t and self.y.

Note: by default this process removes the initial time and state of solution to avoid duplicate times and states being stored (self.t[-1] is equal to solution.t[0], and self.y[:, -1] is equal to solution.y[:, 0]). Set the optional argument start\_index to override this behavior

#### sub\_solutions

List of sub solutions that have been concatenated to form the full solution

### 3.7.8 Post-Process Variables

class pybamm.ProcessedVariable (base\_variable, solution, known\_evals=None, warn=True)

An object that can be evaluated at arbitrary (scalars or vectors) t and x, and returns the (interpolated) value of the base variable at that t and x.

#### **Parameters**

- base\_variable (pybamm. Symbol) A base variable with a method evaluate(t,y) that returns the value of that variable. Note that this can be any kind of node in the expression tree, not just a pybamm. Variable. When evaluated, returns an array of size (m,n)
- **solution** (*pybamm*. *Solution*) The solution object to be used to create the processed variables
- **known\_evals** (dict) Dictionary of known evaluations, to be used to speed up finding the solution

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• warn (bool, optional) - Whether to raise warnings when trying to evaluate time and length scales. Default is True.

```
call_1D(t, x, r, z)
```

Evaluate a 1D variable

### $call_2D(t, x, r, y, z)$

Evaluate a 2D variable

#### data

Same as entries, but different name

```
get_spatial_scale (name, domain)
```

Returns the spatial scale for a named spatial variable

```
initialise_2D()
```

Initialise a 2D object that depends on x and r, or x and z.

```
class pybamm.ProcessedSymbolicVariable (base_variable, solution)
```

An object that can be evaluated at arbitrary (scalars or vectors) t and x, and returns the (interpolated) value of the base variable at that t and x.

#### **Parameters**

- base\_variable (pybamm. Symbol) A base variable with a method evaluate(t,y) that returns the value of that variable. Note that this can be any kind of node in the expression tree, not just a pybamm. Variable. When evaluated, returns an array of size (m,n)
- **solution** (*pybamm*. *Solution*) The solution object to be used to create the processed variables

#### data

Same as entries, but different name

### initialise OD()

Create a 0D variable

#### initialise 1D()

Create a 1D variable

```
sensitivity (inputs=None, check_inputs=True)
```

Returns the sensitivity of the variable to the symbolic inputs at the specified input values

**Parameters** inputs (dict) – The inputs at which to evaluate the variable.

```
value (inputs=None, check_inputs=True)
```

Returns the value of the variable at the specified input values

**Parameters** inputs (dict) – The inputs at which to evaluate the variable.

```
value_and_sensitivity (inputs=None)
```

Returns the value of the variable and its sensitivity to the symbolic inputs at the specified input values

**Parameters** inputs (dict) – The inputs at which to evaluate the variable.

# 3.8 Experiments

Classes to help set operating conditions for some standard battery modelling experiments

# 3.8.1 Base Experiment Class

class pybamm.Experiment (operating\_conditions, parameters=None, period='1 minute')

Base class for experimental conditions under which to run the model. In general, a list of operating conditions should be passed in. Each operating condition should be of the form "Do this for this long" or "Do this until this happens". For example, "Charge at 1 C for 1 hour", or "Charge at 1 C until 4.2 V", or "Charge at 1 C for 1 hour or until 4.2 V". The instructions can be of the form "(Dis)charge at x A/C/W", "Rest", or "Hold at x V". The running time should be a time in seconds, minutes or hours, e.g. "10 seconds", "3 minutes" or "1 hour". The stopping conditions should be a circuit state, e.g. "1 A", "C/50" or "3 V".

#### **Parameters**

- operating\_conditions (list) List of operating conditions
- parameters (dict) Dictionary of parameters to use for this experiment, replacing default parameters as appropriate
- **period** (*string*, *optional*) Period (1/frequency) at which to record outputs. Default is 1 minute. Can be overwritten by individual operating conditions.

#### convert electric (electric)

Convert electrical instructions to consistent output

```
convert_time_to_seconds (time_and_units)
```

Convert a time in seconds, minutes or hours to a time in seconds

```
read_operating_conditions (operating_conditions)
```

Convert operating conditions to the appropriate format

Parameters operating\_conditions (list) - List of operating conditions

Returns operating\_conditions - Operating conditions in the tuple format

Return type list

```
read_string(cond)
```

Convert a string to a tuple of the right format

**Parameters cond** (str) – String of appropriate form for example "Charge at x C for y hours". x and y must be numbers, 'C' denotes the unit of the external circuit (can be A for current, C for C-rate, V for voltage or W for power), and 'hours' denotes the unit of time (can be second(s), minute(s) or hour(s))

# 3.9 Simulation

A Simulation class for easy building and running of PyBaMM simulations.

### **Parameters**

- model (pybamm.BaseModel) The model to be simulated
- **experiment** (pybamm.Experiment (optional)) The experimental conditions under which to solve the model
- **geometry** (pybamm.Geometry (optional)) The geometry upon which to solve the model

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- parameter\_values (pybamm.ParameterValues (optional)) Parameters and their corresponding numerical values.
- submesh\_types (dict (optional)) A dictionary of the types of submesh to use on each subdomain
- var\_pts (dict (optional)) A dictionary of the number of points used by each spatial variable
- **spatial\_methods** (dict (optional)) A dictionary of the types of spatial method to use on each domain (e.g. pybamm.FiniteVolume)
- solver (pybamm. BaseSolver (optional)) The solver to use to solve the model.
- quick\_plot\_vars (list (optional)) A list of variables to plot automatically
- **C\_rate** (*float* (*optional*)) The **C\_rate** at which you would like to run a constant current (dis)charge at.

### build(check\_model=True)

A method to build the model into a system of matrices and vectors suitable for performing numerical computations. If the model has already been built or solved then this function will have no effect. This method will automatically set the parameters if they have not already been set.

**Parameters** check\_model(bool, optional)—If True, model checks are performed after discretisation (see pybamm.Discretisation.process\_model()). Default is True.

#### get\_variable\_array (\*variables)

A helper function to easily obtain a dictionary of arrays of values for a list of variables at the latest timestep.

**Parameters variable** (*str*) – The name of the variable/variables you wish to obtain the arrays for.

**Returns variable\_arrays** – A dictionary of the variable names and their corresponding arrays.

Return type dict

### plot (quick\_plot\_vars=None, testing=False)

A method to quickly plot the outputs of the simulation.

## **Parameters**

- quick\_plot\_vars (list, optional) A list of the variables to plot.
- bool, optional (testing,) If False the plot will not be displayed

### save (filename)

Save simulation using pickle

#### set parameters()

A method to set the parameters in the model and the associated geometry.

# set\_up\_experiment (model, experiment)

Set up a simulation to run with an experiment. This creates a dictionary of inputs (current/voltage/power, running time, stopping condition) for each operating condition in the experiment. The model will then be solved by integrating the model successively with each group of inputs, one group at a time.

**solve** (*t\_eval=None*, *solver=None*, *external\_variables=None*, *inputs=None*, *check\_model=True*)

A method to solve the model. This method will automatically build and set the model parameters if not already done so.

### **Parameters**

• t\_eval (numeric type, optional) - The times (in seconds) at which to compute the solution. Can be provided as an array of times at which to return the solution, or as

a list [t0, tf] where t0 is the initial time and tf is the final time. If provided as a list the solution is returned at 100 points within the interval [t0, tf].

If not using an experiment or running a drive cycle simulation (current provided as data) *t\_eval must* be provided.

If running an experiment the values in  $t_{eval}$  are ignored, and the solution times are specified by the experiment.

If None and the parameter "Current function [A]" is read from data (i.e. drive cycle simulation) the model will be solved at the times provided in the data.

- **solver** (*pybamm.BaseSolver*) The solver to use to solve the model.
- **external\_variables** (dict) A dictionary of external variables and their corresponding values at the current time. The variables must correspond to the variables that would normally be found by solving the submodels that have been made external.
- inputs (dict, optional) Any input parameters to pass to the model when solving
- **check\_model** (bool, optional) If True, model checks are performed after discretisation (see pybamm.Discretisation.process model()). Default is True.
- specs (geometry=None, parameter\_values=None, submesh\_types=None, var\_pts=None, spatial\_methods=None, solver=None, quick\_plot\_vars=None, C\_rate=None)
  Deprecated method for setting specs
- **step** (*dt*, *solver=None*, *npts=2*, *external\_variables=None*, *inputs=None*, *save=True*)

  A method to step the model forward one timestep. This method will automatically build and set the model parameters if not already done so.

#### **Parameters**

- **dt** (numeric type) The timestep over which to step the solution
- solver (pybamm.BaseSolver) The solver to use to solve the model.
- npts (int, optional) The number of points at which the solution will be returned during the step dt. Default is 2 (returns the solution at t0 and t0 + dt).
- **external\_variables** (dict) A dictionary of external variables and their corresponding values at the current time. The variables must correspond to the variables that would normally be found by solving the submodels that have been made external.
- inputs (dict, optional) Any input parameters to pass to the model when solving
- save (bool) Turn on to store the solution of all previous timesteps

# 3.10 Plotting

### 3.10.1 Quick Plot

```
class pybamm.QuickPlot (solutions, output_variables=None, labels=None, colors=None, linestyles=None, figsize=None, time_unit=None, spatial_unit='um', variable_limits='fixed')
```

Generates a quick plot of a subset of key outputs of the model so that the model outputs can be easily assessed.

#### **Parameters**

• **solutions** ((iter of) *pybamm.Solution* or *pybamm.Simulation*) – The numerical solution(s) for the model(s), or the simulation object(s) containing the solution(s).

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- output\_variables (list of str, optional) List of variables to plot
- labels (list of str, optional) Labels for the different models. Defaults to model names
- **colors** (list of str, optional) The colors to loop over when plotting. Defaults to ["r", "b", "k", "g", "m", "c"]
- linestyles (list of str, optional) The linestyles to loop over when plotting. Defaults to ["-", ":", "-", "-."]
- figsize (tuple of floats, optional) The size of the figure to make
- time\_unit (str, optional) Format for the time output ("hours", "minutes" or "seconds")
- spatial\_unit (str, optional) Format for the spatial axes ("m", "mm" or "um")
- variable\_limits (str or dict of str, optional) How to set the axis limits (for 0D or 1D variables) or colorbar limits (for 2D variables). Options are:
  - "fixed" (default): keep all axes fixes so that all data is visible
  - "tight": make axes tight to plot at each time
  - dictionary: fine-grain control for each variable, can be either "fixed" or "tight" or a specific tuple (lower, upper).

#### dynamic\_plot (testing=False, step=None)

Generate a dynamic plot with a slider to control the time.

#### **Parameters**

- **step** (*float*) For notebook mode, size of steps to allow in the slider. Defaults to 1/100th of the total time.
- **testing** (bool) Whether to actually make the plot (turned off for unit tests)

### get\_spatial\_var(key, variable, dimension)

Return the appropriate spatial variable(s)

#### plot(t)

Produces a quick plot with the internal states at time t.

**Parameters** t (float) – Dimensional time (in 'time units') at which to plot.

### reset axis()

Reset the axis limits to the default values. These are calculated to fit around the minimum and maximum values of all the variables in each subplot

#### slider update(t)

Update the plot in self.plot() with values at new time

# 3.10.2 Dynamic Plot

```
pybamm.dynamic_plot (*args, **kwargs)
```

Creates a pybamm.QuickPlot object (with arguments 'args' and keyword arguments 'kwargs') and then calls pybamm.QuickPlot.dynamic\_plot(). The key-word argument 'testing' is passed to the 'dynamic\_plot' method, not the QuickPlot class.

**Returns** plot – The 'QuickPlot' object that was created

Return type pybamm.QuickPlot

# 3.10.3 Plot

pybamm.plot(x, y, xlabel=None, ylabel=None, title=None, testing=False, \*\*kwargs)

Generate a simple 1D plot. Calls *matplotlib.pyplot.plot* with keyword arguments 'kwargs'. For a list of 'kwargs' see the matplotlib plot documentation

#### **Parameters**

- **x** (pybamm.Array) The array to plot on the x axis
- y (pybamm. Array) The array to plot on the y axis
- xlabel (str, optional) The label for the x axis
- ylabel (str, optional) The label for the y axis
- **testing** (bool, optional) Whether to actually make the plot (turned off for unit tests)

### 3.10.4 Plot 2D

pybamm.plot2D (x, y, z, xlabel=None, ylabel=None, title=None, testing=False, \*\*kwargs)

Generate a simple 2D plot. Calls *matplotlib.pyplot.contourf* with keyword arguments 'kwargs'. For a list of 'kwargs' see the matplotlib contourf documentation

#### **Parameters**

- x (pybamm. Array) The array to plot on the x axis. Can be of shape (M, N) or (N, 1)
- y (pybamm. Array) The array to plot on the y axis. Can be of shape (M, N) or (M, 1)
- **z** (pybamm. Array) The array to plot on the z axis. Is of shape (M, N)
- xlabel (str, optional) The label for the x axis
- ylabel (str, optional) The label for the y axis
- title (str, optional) The title for the plot
- testing (bool, optional) Whether to actually make the plot (turned off for unit tests)

# 3.11 Utility functions

```
pybamm.get_infinite_nested_dict()
```

Return a dictionary that allows infinite nesting without having to define level by level.

See: https://stackoverflow.com/questions/651794/whats-the-best-way-to-initialize-a-dict-of-dicts-in-python/652226#652226

# **Example**

```
>>> import pybamm
>>> d = pybamm.get_infinite_nested_dict()
>>> d["a"] = 1
>>> d["a"]
1
>>> d["b"]["c"]["d"] = 2
```

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```
>>> d["b"]["c"] == {"d": 2}
True
```

```
pybamm.load_function(filename)
```

Load a python function from a file "function\_name.py" called "function\_name". The filename might either be an absolute path, in which case that specific file will be used, or the file will be searched for relative to PyBaMM root.

**Parameters filename** (str) – The name of the file containing the function of the same name.

**Returns** The python function loaded from the file.

Return type function

```
pybamm.rmse(x, y)
```

Calculate the root-mean-square-error between two vectors x and y, ignoring NaNs

```
pybamm.root_dir()
```

return the root directory of the PyBaMM install directory

#### class pybamm.Timer

Provides accurate timing.

### **Example**

```
timer = pybamm.Timer() print(timer.format(timer.time()))
```

```
format (time=None)
```

Formats a (non-integer) number of seconds, returns a string like "5 weeks, 3 days, 1 hour, 4 minutes, 9 seconds", or "0.0019 seconds".

**Parameters time** (float, optional) – The time to be formatted.

**Returns** The string representation of time in human-readable form.

Return type string

#### reset()

Resets this timer's start time.

```
time()
```

Returns the time (float, in seconds) since this timer was created, or since meth: reset() was last called.

# 3.12 Citations

### class pybamm.Citations

Entry point to citations management. This object may be used to record Bibtex citation information and then register that a particular citation is relevant for a particular simulation. For a list of all possible citations, see *pybamm/CITATIONS.txt* 

### **Examples**

```
>>> import pybamm
>>> pybamm.citations.register("sulzer2020python")
>>> pybamm.print_citations("citations.txt")
```

```
print (filename=None)
```

Print all citations that were used for running simulations.

**Parameters filename** (*str*, *optional*) – Filename to which to print citations. If None, citations are printed to the terminal.

```
read citations()
```

Read the citations text file

```
register (key)
```

Register a paper to be cited. The intended use is that register() should be called only when the referenced functionality is actually being used.

**Parameters key** (str) – The key for the paper to be cited

```
pybamm.print_citations (filename=None)
    See Citations.print()
```

# 3.13 Parameters command line interface

PyBaMM comes with a small command line interface that can be used to manage parameter sets. By default, PyBaMM provides parameters in the "input" directory located in the pybamm package directory. If you wish to add new parameters, you can first pull a given parameter directory into the current working directory using the command pybamm\_edit\_parameter for manual editing. By default, PyBaMM first looks for parameter defined in the current working directory before falling back the package directory if nothing is found locally. If you wish to access a newly defined parameter set from anywhere in your system, you can use pybamm\_add\_parameter to copy a given parameter directory to the package directory. To get a list of currently available parameter sets, use pybamm\_list\_parameters.

```
pybamm.parameters_cli.add_parameter(arguments=None)
```

Add a parameter directory to package input directory. This allows the parameters to be used from anywhere in the system.

Example: "add\_parameter foo lithium-ion anodes" will copy directory foo in "pybamm/input/parameters/lithium-ion/anodes".

```
pybamm.parameters_cli.remove_parameter(arguments=None)
```

Remove a parameter directory from package input directory.

Example: "rm\_parameter foo lithium-ion anodes" will remove directory foo in "pybamm/input/parameters/lithium-ion/anodes".

```
pybamm.parameters_cli.edit_parameter(arguments=None)
```

Copy a given default parameter directory to the current working directory for editing. For example

```
edit_param(["lithium-ion"])
```

will create the directory structure:

```
lithium-ion/
anodes/
graphite_Chen2020
...
cathodes/
...
```

in the current working directory.

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# Examples

 $\label{eq:camples} Detailed \ examples \ can \ be \ viewed \ on \ the \ GitHub \ examples \ page, \ and \ run \ locally \ using \ \verb"jupyter" \ notebook, or online through Binder.$ 

Contributing

There are many ways to contribute to PyBaMM:

# 5.1 Adding Parameter Values

As with any contribution to PyBaMM, please follow the workflow in CONTRIBUTING.md. In particular, start by creating an issue to discuss what you want to do - this is a good way to avoid wasted coding hours!

# 5.1.1 The role of parameter values

All models in PyBaMM are implemented as expression trees. At the stage of creating a model, we use pybamm. Parameter and pybamm. FunctionParameter objects to represent parameters and functions respectively.

We then create a ParameterValues class, using a specific set of parameters, to iterate through the model and replace any pybamm. Parameter objects with a pybamm. Scalar and any pybamm. FunctionParameter objects with a pybamm. Function.

For an example of how the parameter values work, see the parameter values notebook.

# 5.1.2 Adding a set of parameters values

Parameter sets are split by material into anodes, separators, cathodes, electrolytes, cells (for cell geometries and thermal properties) and experiments (for initial conditions and charge/discharge rates). To add a new parameter set in one of these subcategories, first create a new folder in the appropriate chemistry folder: for example, to add a new anode chemistry for lithium-ion, add a subfolder input/parameters/lithium-ion/anodes/new\_anode\_chemistry\_AuthorYear. This subfolder should then contain:

• a csv file parameters.csv with all the relevant scalar parameters. The expected structure of the csv file is:

Name [Units]	Value	Reference	Notes
Example [m]	13	AuthorYear	an example

Empty lines, and lines starting with #, will be ignored.

- a README . md file with information on where these parameters came from
- python files for any functions, which should be referenced from the parameters.csv file (see Adding a Function below)
- csv files for any data to be interpolated, which should be referenced from the parameters.csv file (see Adding data for interpolation below)

The easiest way to start is to copy an existing file (e.g. ``input/parameters/lithium-ion/anodes/graphite\_mcmb2528\_Marquis2019) and replace all entries in all files as appropriate

# 5.1.3 Adding a function

Functions should be added as Python functions under a file with the same name in the appropriate chemistry folder in input/parameters/. These Python functions should be documented with references explaining where they were obtained. For example, we would put the following Python function in a file input/parameters/lithium\_ion/anodes/new\_anode\_chemistry\_AuthorYear/diffusivity\_AuthorYear.py

Then, these functions should be added to the parameter file from which they will be called (must be in the same folder), with the tag [function], for example:

Name [Units]	Value	Reference	Notes
Example [m2.s-1]	[function]diffusivity_AuthorYear	AuthorYear	a function

# 5.1.4 Adding data for interpolation

Data should be added as as csv file in the appropriate chemistry folder in input/parameters/. For example, we would put the following data in a file input/parameters/lithium\_ion/anodes/new\_anode\_chemistry\_AuthorYear/diffusivity\_AuthorYear.csv

# concentration [r	mol/m3]	Diffusivity [m2/s]
0.0000000000000000000000000000000000000	0000e+00	4.714135898019971016e+00
2.04081632653061	12082e-	4.708899441575220557e+00
02	4.081632653061224164e-	4.702448345762175741e+00
02	6.122448979591836593e-	4.694558534379876136e+00
02	8.163265306122448328e-	4.684994372928071193e+00
02	1.020408163265306006e-	4.673523893805322516e+00
01	1.224489795918367319e-01	4.659941254449398329e+00
1.42857142857142	28492e-01	4.644096031712390271e+00

Empty lines, and lines starting with #, will be ignored.

Then, this data should be added to the parameter file from which it will be called (must be in the same folder), with the tag [data], for example:

Name [Units]	Value	Reference	Notes
Example [m2.s-1]	[data]diffusivity_AuthorYear	AuthorYear	some data

# 5.1.5 Using new parameters

If you have added a whole new set of parameters, then you can create a new parameter set in pybamm/parameters/parameter\_sets.py, by just adding a new dictionary to that file, for example

```
AuthorYear = {
    "chemistry": "lithium-ion",
    "cell": "new_cell_AuthorYear",
    "anode": "new_anode_AuthorYear",
    "separator": "new_separator_AuthorYear",
    "cathode": "new_cathode_AuthorYear",
    "electrolyte": "new_electrolyte_AuthorYear",
    "experiment": "new_experiment_AuthorYear",
}
```

Then, to use these new parameters, use:

```
param = pybamm.ParameterValues(chemistry=pybamm.parameter_sets.AuthorYear)
```

Note that you can re-use existing parameter subsets instead of creating new ones (for example, you could just replace "experiment": "new\_experiment\_AuthorYear" with "experiment": "1C\_discharge\_from\_full\_Marquis2019" in the above dictionary).

It's also possible to add parameters for a single material (e.g. anode) and then re-use existing parameters for the other materials, without adding a parameter set to pybamm/parameters/parameter\_sets.py.

```
param = pybamm.ParameterValues(
    chemistry={
        "chemistry": "lithium-ion",
        "cell": "kokam_Marquis2019",
        "anode": "new_anode_chemistry_AuthorYear",
        "separator": "separator_Marquis2019",
        "cathode": "lico2_Marquis2019",
        "electrolyte": "lipf6_Marquis2019",
        "experiment": "1C_discharge_from_full_Marquis2019",
    }
}
```

or, equivalently in this case (since the only difference from the standard parameters from Marquis et al. is the set of anode parameters),

```
param = pybamm.ParameterValues(
    chemistry={
        **pybamm.parameter_sets.Marquis2019,
        "anode": "new_anode_chemistry_AuthorYear",
    }
)
```

See the "Getting Started" tutorial for examples of setting parameters in action.

### 5.1.6 Unit tests for the new class

You might want to add some unit tests to show that the parameters combine as expected (see e.g. lithium-ion parameter tests), but this is not crucial.

### 5.1.7 Test on the models

In theory, any existing model can now be solved using the new parameters instead of their default parameters, with no extra work from here. To test this, add something like the following test to one of the model test files (e.g. DFN):

```
def test_my_new_parameters(self):
    model = pybamm.lithium_ion.DFN()
    parameter_values = pybamm.ParameterValues(chemistry=pybamm.parameter_sets.
    →AuthorYear)
    modeltest = tests.StandardModelTest(model, parameter_values=parameter_values)
    modeltest.test_all()
```

This will check that the model can run with the new parameters (but not that it gives a sensible answer!).

Once you have performed the above checks, you are almost ready to merge your code into the core PyBaMM - see CONTRIBUTING.md workflow for how to do this.

# 5.2 Adding a Model

As with any contribution to PyBaMM, please follow the workflow in CONTRIBUTING.md. In particular, start by creating an issue to discuss what you want to do - this is a good way to avoid wasted coding hours!

We aim here to provide an overview of how a new model is entered into PyBaMM in a form which can be eventually merged into the master branch of the PyBaMM project. However, we recommend that you first read through the notebook: create a model, which goes step-by-step through the procedure for creating a model. Once you understand that procedure, you can then formalise your model following the outline provided here.

#### 5.2.1 The role of models

One of the main motivations for PyBaMM is to allow for new models of batteries to be easily be added, solved, tested, and compared without requiring a detailed knowledge of sophisticated numerical methods. It has therefore been our focus to make the process of adding a new model as simple as possible. To achieve this, all models in PyBaMM are implemented as expression trees, which abstract away the details of computation.

The fundamental building blocks of a PyBaMM expression tree are pybamm. Symbol. There are different types of pybamm. Symbol: pybamm. Variable, pybamm. Parameter, pybamm. Addition, pybamm. Multiplication, pybamm. Gradient etc which have been created so that each component of a model written out in PyBaMM mirrors exactly the written mathematics. For example, the expression:

$$\nabla \cdot (D(c)\nabla c) + aFj$$

is simply written as

```
div(D(c) * grad(c)) + a * F * j
```

within PyBaMM. A model in PyBaMM is essentially an organised collection of expression trees.

### 5.2.2 Implementing a new model

To add a new model (e.g. My New Model), first create a new file (my\_new\_model.py) in pybamm/models (or the relevant subdirectory). In this file create a new class which inherits from pybamm.BaseModel (or pybamm.LithiumIonBaseModel if you are modelling a full lithium-ion battery or pybamm.LeadAcidBaseModel if you are modelling a full lead acid battery):

```
class MyNewModel(pybamm.BaseModel):
    def
```

and add the class to pybamm/\_\_init\_\_.py:

```
from .models.my_new_model import MyNewModel
```

(this line will be slightly different if you created your model in a subdirectory of models). Within your new class MyNewModel, first create an initialisation function which calls the initialisation function of the parent class

```
def __init__(self):
    super().__init__()
```

Within the initialisation function of MyNewModel you must then define the following attributes:

- self.rhs
- self.algebraic
- self.boundary\_conditions
- self.initial conditions
- self.variables

You may also optionally also provide:

- self.events
- self.default\_geometry
- self.default\_solver
- self.default\_spatial\_methods
- self.default\_submesh\_types
- self.default\_var\_pts
- self.default\_parameter\_values

We will go through each of these attributes in turn here for completeness but refer the user to the API documentation or example notebooks (create-model.ipnb) if further details are required.

#### Governing equations

The governing equations which can either be parabolic or elliptic are entered into the self.rhs and self. algebraic dictionaries, respectively. We associate each governing equation with a subject variable, which is the variable that is found when the equation is solved. We use this subject variable as the key of the dictionary. For parabolic equations, we rearrange the equation so that the time derivative of the subject variable is the only term on the left hand side of the equation. We then simply write the resulting right hand side into the self.rhs dictionary with the subject variable as the key. For elliptic equations, we rearrange so that the left hand side of the equation if zero and then write the right hand side into the self.algebraic dictionary in the same way. The resulting dictionary should look like:

```
self.rhs = {parabolic_var1: parabolic_rhs1, parabolic_var2: parabolic_rhs2, ...}
self.algebraic = {elliptic_var1: elliptic_rhs1, elliptic_var2: elliptic_rhs2, ...}
```

#### **Boundary conditions**

Boundary conditions on a variable can either be Dirichlet or Neumann (support for mixed boundary conditions will be added at a later date). For a variable c on a one dimensional domain with a Dirichlet condition of c=1 on the left boundary and a Neumann condition of  $\nabla c=2$  on the right boundary, we then have:

```
self.boundary_conditions = {c: {"left": (1, "Dirichlet"), "right": (2, "Neumann")}}
```

#### **Initial conditions**

For a variable c that is initially at a value of c = 1, the initial condition is included written into the model as

```
self.initial_conditions = {c: 1}
```

#### **Output variables**

PyBaMM allows users to create combinations of symbols to output from their model. For example, we might wish to output the terminal voltage which is given by  $V = \phi_{s,p}|_{x=1} - \phi_{s,n}|_{x=0}$ . We would first define the voltage symbol V and then include it into the output variables dictionary in the form:

```
self.variables = {"Terminal voltage [V]": V}
```

Note that we indicate that the quantity is dimensional by including the dimensions, Volts in square brackets. We do this to distinguish between dimensional and dimensionless outputs which may otherwise share the same name.

Note that if your model inherits from pybamm. StandardBatteryBaseModel, then there is a standard set of output parameters which is enforced to ensure consistency across models so that they can be easily tested and compared.

#### **Events**

Events can be added to stop computation when the event occurs. For example, we may wish to terminate our computation when the terminal voltage V reaches some minimum voltage during a discharge  $V_{min}$ . We do this by adding the following to the events dictionary:

```
self.events["Minimum voltage cut-off"] = V - V_min
```

Events will stop the solver whenever they return 0.

#### **Setting defaults**

It can be useful for testing, and quickly running a model to have a default setup. Each of the defaults listed above should adhere to the API requirements but in short, we require self.default\_geometry to be a dictionary of the right format (see <a href="pybamm.battery\_geometry">pybamm.battery\_geometry</a>()), self.default\_solver to be an instance of <a href="pybamm.BaseSolver">pybamm.BaseSolver</a>, and self.default\_parameter\_values to be an instance of <a href="pybamm.ParameterValues">pybamm.ParameterValues</a>. We also require that self.default\_submesh\_types is a dictionary with keys which are strings corresponding to the regions of the battery (e.g. "negative electrode") and values which are an instance

of pybamm. SubMesh1D. The self.default\_spatial\_methods attribute is also required to be a dictionary with keys corresponding to the regions of the battery but with values which are an instance of pybamm. SpatialMethod. Finally, self.default\_var\_pts is required to be a dictionary with keys which are an instance of pybamm. SpatialVariable and values which are integers.

#### Using submodels

The inbuilt models in PyBaMM do not add all the model attributes within their own file. Instead, they make use of inbuilt submodel (a particle model, an electrolyte model, etc). There are two main reasons for this. First, the code in the submodels can then be used by multiple models cutting down on repeated code. This makes it easier to maintain the codebase because fixing an issue in a submodel fixes that issue everywhere the submodel is called (instead of having to track down the issue in every model). Secondly, it allows for the user to easily switch a submodel out for another and study the effect. For example, we may be using standard diffusion in the particles but decide that we would like to switch in particles which are phase separating. With submodels all we need to do is switch the submodel instead of re-writing the whole sections of the model. Submodel contributions are highly encouraged so where possible, try to divide your model into submodels.

In addition to calling submodels, common sets of variables and parameters found in lithium-ion and lead acid batteries are provided in *standard\_variables.py*, *standard\_parameters\_lithium\_ion.py*, *standard\_parameters\_lead\_acid.py*, *electrical\_parameters.py*, *geometric\_parameters.py*, and *standard\_spatial\_vars.py* which we encourage use of to save redefining the same parameters and variables in every model and submodel.

# 5.2.3 Unit tests for a MyNewModel

We strongly recommend testing your model to ensure that it is behaving correctly. To do this, first create a new file test\_my\_new\_model.py within tests/integration/test\_models (or the appropriate subdirectory). Within this file, add the following code

```
import pybamm
import unittest

class TestMyNewModel(unittest.TestCase):
    def my_first_test(self):
        # add test here

if __name__ == "__main__":
    print("Add -v for more debug output")
    import sys

if "-v" in sys.argv:
    debug = True
    unittest.main()
```

We can now add functions such as my\_first\_test() to TestMyNewModel which run specific tests. As a first test, we recommend you make use of tests.StandardModelTest which runs a suite of basic tests. If your new model is a full model of a battery and therefore inherits from pybamm.StandardBatteryBaseModel then tests.StandardBatteryTest will also check the set of outputs are producing reasonable behaviour.

Please see the tests of the inbuilt models to get a further idea of how to test the your model.

# 5.3 Adding a Spatial Method

As with any contribution to PyBaMM, please follow the workflow in CONTRIBUTING.md. In particular, start by creating an issue to discuss what you want to do - this is a good way to avoid wasted coding hours!

# 5.3.1 The role of spatial methods

All models in PyBaMM are implemented as expression trees. After it has been created and parameters have been set, the model is passed to the *pybamm.Discretisation* class, which converts it into a linear algebra form. For example, the object:

```
grad(u)
```

might get converted to a Matrix-Vector multiplication:

```
Matrix(100,100) @ y[0:100]
```

(in Python 3.5+, @ means matrix multiplication, while \* is elementwise product). The pybamm.Discretisation class is a wrapper that iterates through the different parts of the model, performing the trivial conversions (e.g. Addition -> Addition), and calls upon spatial methods to perform the harder conversions (e.g. grad(u) -> Matrix \* StateVector, SpatialVariable -> Vector, etc).

Hence SpatialMethod classes only need to worry about the specific conversions, and pybamm. Discretisation deals with the rest.

# 5.3.2 Implementing a new spatial method

To add a new spatial method (e.g. My Fast Method), first create a new file (my\_fast\_method.py) in pybamm/spatial\_methods/, with a single class that inherits from pybamm. SpatialMethod, such as:

```
class MyFastMethod(pybamm.SpatialMethod):
```

and add the class to pybamm/\_\_init\_\_.py:

```
from .spatial_methods.my_fast_method import MyFastMethod
```

You can then start implementing the spatial method by adding functions to the class. In particular, any spatial method *must* have the following functions (from the base class <code>pybamm.SpatialMethod</code>):

- pybamm. Spatial Method. gradient ()
- pybamm. Spatial Method. divergence ()
- pybamm. Spatial Method. integral ()
- pybamm.SpatialMethod.indefinite integral()
- pybamm.SpatialMethod.boundary\_value\_or\_flux()

Optionally, a new spatial method can also overwrite the default behaviour for the following functions:

- pybamm.SpatialMethod.spatial\_variable()
- pybamm.SpatialMethod.broadcast()
- pybamm.SpatialMethod.mass\_matrix()
- pybamm.SpatialMethod.process\_binary\_operators()

• pybamm. Spatial Method. concatenation ()

For an example of an existing spatial method implementation, see the Finite Volume API docs and notebook.

#### 5.3.3 Unit tests for the new class

For the new spatial method to be added to PyBaMM, you must add unit tests to demonstrate that it behaves as expected (see, for example, the Finite Volume unit tests). The best way to get started would be to create a file test\_my\_fast\_method.py in tests/unit/test\_spatial\_methods/ that performs at least the following checks:

- Operations return objects that have the expected shape
- Standard operations behave as expected, e.g. (in 1D)  $grad(x^2) = 2*x$ , integral(sin(x), 0, pi) = 2
- (more advanced) make sure that the operations converge at the correct rate to known analytical solutions as you decrease the grid size

#### 5.3.4 Test on the models

In theory, any existing model can now be discretised using MyFastMethod instead of their default spatial methods, with no extra work from here. To test this, add something like the following test to one of the model test files (e.g. DFN):

```
def test_my_fast_method(self):
    model = pybamm.lithium_ion.DFN()
    spatial_methods = {
        "macroscale": pybamm.MyFastMethod,
        "negative particle": pybamm.MyFastMethod,
        "positive particle": pybamm.MyFastMethod,
    }
    modeltest = tests.StandardModelTest(model, spatial_methods=spatial_methods)
    modeltest.test_all()
```

This will check that the model can run with the new spatial method (but not that it gives a sensible answer!).

Once you have performed the above checks, you are almost ready to merge your code into the core PyBaMM - see CONTRIBUTING.md workflow for how to do this.

# 5.4 Adding a Solver

As with any contribution to PyBaMM, please follow the workflow in CONTRIBUTING.md. In particular, start by creating an issue to discuss what you want to do - this is a good way to avoid wasted coding hours!

#### 5.4.1 The role of solvers

All models in PyBaMM are implemented as expression trees. After the model has been created, parameters have been set, and the model has been discretised, the model is now a linear algebra object with the following attributes:

model.concatenated\_rhs A pybamm. Symbol node that can be evaluated at a state (t, y) and returns the value of all the differential equations at that state, concatenated into a single vector

model.concatenated\_algebraic A pybamm.Symbol node that can be evaluated at a state (t, y) and returns the value of all the algebraic equations at that state, concatenated into a single vector

model.concatenated\_initial\_conditions A numpy array of initial conditions for all the differential and algebraic equations, concatenated into a single vector

model.events A dictionary of pybamm. Symbol nodes representing events at which the solver should terminate. Specifically, the solver should terminate when any of the events in model.events.values() evaluate to zero

The role of solvers is to solve a model at a given set of time points, returning a vector of times t and a matrix of states y.

### 5.4.2 Base solver classes vs specific solver classes

There is one general base solver class, <code>pybamm.BaseSolver</code>, which sets up some useful solver properties such as tolerances and implement a method <code>self.solve()</code> that solves a model at a given set of time points.

The solve method unpacks the model, simplifies it by removing extraneous operations, (optionally) creates or calls the mass matrix and/or jacobian, and passes the appropriate attributes to another method, called integrate, which does the time-stepping. The role of specific solver classes is simply to implement this integrate method for an arbitrary set of derivative function, initial conditions etc.

The base solver class also computes a consistent set of initial conditions for the algebraic equations, using model. concatenated\_initial\_conditions as an initial guess.

## 5.4.3 Implementing a new solver

To add a new solver (e.g. My Fast DAE Solver), first create a new file (my\_fast\_dae\_solver.py) in pybamm/solvers/, with a single class that inherits from pybamm. BaseSolver. For example:

```
def MyFastDaeSolver(pybamm.BaseSolver):
```

Also add the class to pybamm/\_\_init\_\_.py:

```
from .solvers.my_fast_dae_solver import MyFastDaeSolver
```

You can then start implementing the solver by adding the integrate function to the class.

For an example of an existing solver implementation, see the Scikits DAE solver API docs and notebook.

#### 5.4.4 Unit tests for the new class

For the new solver to be added to PyBaMM, you must add unit tests to demonstrate that it behaves as expected (see, for example, the Scikits solver tests). The best way to get started would be to create a file test\_my\_fast\_solver.py in tests/unit/test\_solvers/ that performs at least the following checks:

- The integrate method works on a simple ODE/DAE model with/without jacobian, mass matrix and/or events as appropriate
- The solve method works on a simple model (in theory, if the integrate method works then the solve method should always work)

If the solver is expected to converge in a certain way as the time step is changed, you could also add a convergence test in tests/convergence/solvers/.

### 5.4.5 Test on the models

In theory, any existing model can now be solved using *MyFastDaeSolver* instead of their default solvers, with no extra work from here. To test this, add something like the following test to one of the model test files (e.g. DFN):

```
def test_my_fast_solver(self):
    model = pybamm.lithium_ion.DFN()
    solver = pybamm.MyFastDaeSolver()
    modeltest = tests.StandardModelTest(model, solver=solver)
    modeltest.test_all()
```

This will check that the model can run with the new solver (but not that it gives a sensible answer!).

Once you have performed the above checks, you are almost ready to merge your code into the core PyBaMM - see CONTRIBUTING.md workflow for how to do this.

Before contributing, please read the Contribution Guidelines.

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